Application of Graph Theory in Scheduling Tournament

Berdewad OK and Deo SD
Department of Mathematics, NES College, Bhadrawati, Dist Chandrapur, India.
Gondwana University, Gadachiroli, MS, India.

Abstract:

The field of mathematics plays an important role in various field, one of the important areas in mathematics is graph theory. The present paper states that various application of graph theory in scheduling a tournament, computer sciences, networking & an overview has been presented here.

1.Introduction:-

Sports tournament are main economic activities around the world. They draw attention of millions of people aero the globe. The broadcasters and organizer invest a lot of money in sports events. The schedule is the main aspect of the tournament. On one hand, there are multiple decision makers; the broadcasters, the team, the organizer and the government. For there quandary of scheduling. Sports tournament have gained considerable amount of attention in recent years among the operation research perpetually.

Keywords: Graph, vertices, edges, directed path, Hamiltonian path, and scheduling tournament

2.Some Basic definitions of Graph theory

a) Graph:- A group is an ordered pair G=(V,E) comprising a set of vertices (or nodes) together with a set E of edges (or line).

b) Vertices:- If G=(V,E) is a graph then the set V is said to vertex set of a graph G and the member V are said to be vertices of the graph.

c) Edges:- The family E is said to be edge of the graph G and the member of E is said to be the edges of the graph G.

d) Directed graph or digraph: A directed graph or digraph is an ordered pair D=(V,E) where each edge has a direction.[2]

e) Complete graph: A complete graph is a simple graph in which each pair of distinct vertices is joined by edges.

3.Some properties of Tournaments

If every team has its own home field it is desirable to schedule the tournament in such way that the home and away games for every team alternate as regularly as possible. A team has a break in the schedule when it plays two successive home or away games. The most balanced schedule is one with no breaks. Suppose that there are at least three teams. Then either at

International Journal of Scientific Engineering and Applied Science (IJSEAS) - Volume-1, Issue-5, August 2015
ISSN: 2395-3470
www.ijseas.com

least two of them starts their Schedule with a home game, or at least two of them start with an away game without loss of generality. We can suppose that two starts with a home game, but since both of them have no breaks, they always play home at the same time and they never play each other.

A Kirkman tournament has the property that its rounds can be recorded so that two teams have no breaks while all other teams have precisely one break each.

Ex. Use the convection that in a game k-j the home team is j and schedule round 1 as ∞-1, 7-2, 6-3, 5-4. Round 2 is obtained from round 1 by adding 4 to each team number. In general, Round (i+1) is obtained from round i by adding 4 to each team number and the games involving team ∞ alternate ∞ as the away and home team. Then team ∞ and 4 have no breaks teams 1, 2 and 3 have one home break each and team 5, 6 and 7 have one way break each. If we want to be fair and have one break for each team, we can schedule the games between ∞ and 4 in the round 7 as 4-∞ rather than ∞-4. Thus also works for 2n teams and round i+1 is then obtained from round I by adding n to each team number.

We want to schedule a tournament for an odd number of team 2n-1, we can take any schedule for 2n teams and pick a team j to be the dummy team. Whatever team is schedule to play the dummy team in round i then is said to have a bye in that round this is the only schedule for odd number of teams with

this property as discovered by Mariusz Meszka and the author [7], who proved that for only even number of teams there exist a unique schedule in which every team has one bye and no break.

The main aspect of round robin tournament is the carry over effect [8]. When there are games k-j in round i and k-t in round (i+1). We say that team t received carry over effect team j in round (i+1) Kirkman Tournament as above properly, we can see that team 1 receives carry over effect from team 6 in 5 rounds, namely n rounds 2 though 6 and once from team ∞ in round 7 i.e., team 1 plays six times during the tournament against the team that played team 6 in the previous round. Thus may be an advantage or disadvantage depending on weather team 6 is the best or not best team in the tournament. Similarly all team except ∞ receives the carry over effect from the same team five times while ∞ receives it in each round from different team this is due to the rotational structure of Kirkman tournament and the special role of ∞ in it. There are tournament that have perfect carry over effect. i.e., every team receives the carry over effect from each other team at most once. However such a tournament are known to exist only when the number of teams is either a power of two [8], 22 or 22[1]. Unfortunately, the tournaments with perfect carry over effect typically have very bad break structures and balancing both properties are difficult. Some example of league where both properties are well balanced can be found in [5].

International Journal of Scientific Engineering and Applied Science (IJSEAS) - Volume-1, Issue-5, August 2015
ISSN: 2395-3470
www.ijseas.com

4.Example : Let T={1,2,3,4,5,6}be the set contestants in the initial tournament and Tr={1,2,3,4}be the set of containsts to be removed. Thus n=6, n^r=4.

case1: Let r=3 so that r>n-nr=2.In what follows. We consider a schedule where maximum number of matches involving the removed contestants is kept after round r=3

The result of a round robin tournament can be represented with a tournament graph. This is a directed graph in which the vertices and edges represents players outcome respectively the games. In particular, an edges from x to y indicates that players x detected y. In a round robin tournament, every pair of players has a match. Thus in a tournament graph there is a either an edges from x to y or an edges from y to x every pair of vertices x to y. Here is an example of a tournament graph.[3]

Rounds	Match 1	Match 2	Match 3
Round 1	2 √ 4	3 √ 1	5 √ 6
Round 2	1 √ 5	6 √ 4	2 √ 3
Round 3	3 √ 6	1 √ 2	4 √ 5
Round 4	6 √ 2	5 √ 3	1 √ 4
Round 5	4 √ 3	2 √ 5	6 √ 1

5) Tournament Rankings:-

Suppose that n players complete in a round-robin tournament. Thus, for every pair of players x and y either x beats y or else y beats x. Interpreting the results of a round robin tournament can be problematizes. There might be all sorts of cycles where p beat q, q beat r yet r beat p. Graph theory provides at minimum solution of this problem.[4]
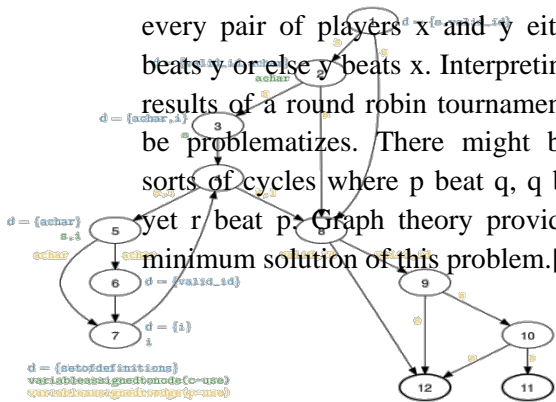
The another notation of walks, Euler tours and Hamiltonian cycles all carry over naturally to directed graphs. A

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International Journal of Scientific Engineering and Applied Science (IJSEAS) - Volume-1, Issue-5, August 2015
ISSN: 2395-3470
www.ijseas.com

directed walk is an alternating sequence of vertices and directed edges: $v_0, v_0 \rightarrow v_1, v_1, v_1 \rightarrow v_2, v_2$_____$v_{n-1}, v_n, v_n$.

A directed Hamiltonian path is directed walk that visits every vertex exactly once. We are going to prove that in every round robin tournament, there exist a ranking of the player such that player lost to the player ranking one position higher. For example, in the tournament above, the rankings.A>B>D>E>C Satisfies this criterion, because B lost to A,D lost to B,E lost to B,E lost to D and C lost to E. in graph teams providing the existence of such a ranking amounts to proving that every tournament graph has a Hamiltonian path.[3]

Theorem: Every tournament graph contains a directed Hamiltonian path.

Proof: We use strong induction. Let $\rho(n)$ be the preposition that every tournament graph with n vertices contains a directed Hamiltonian path.

Base case: $\rho(1)$ is trivially true, every graph with a single vertex has a Hamiltonian path consisting of only of only that vertex.

Inductive step: For n=1, we assume that $\rho(1)$--------$\rho(n)$ are also true and prove and (n+1). Consider a tournament with (n+1) players. Select one vertex v arbitrary. Every other vertex in the tournament either has an edges to vertex v or an edges from vertex v. thus, we can partition the remaining the vertices into two corresponding set, T and E each containing at most n vertices.

The vertices in T together with the edges that join them form a smaller tournament. Thus, by small induction, there is a Hamiltonian path within T. similarly; there is a Hamiltonian path within the tournament on the vertices in the F. joining the path T to the vertex followed by the path in F gives a Hamiltonian path through the whole tournament. (as a special case if T or F is empty, them so is the corresponding portion of path).

The ranking defined by a Hamiltonian path is not entirely satisfactorily. In the example tournament, notice that the lowest ranked player (C) actually denoted by the highest ranked player (A)!

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