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On the Multiplication of Identical Numbers to Summation of its Lagged Multiplicands

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Abstract

Multiplication of Identical Numbers (MIN) was considered to determine the model that performs better to ease the complicated concept of multiplication. Properties of multiplication were examined with core interest on identity, commutative and distributive laws of identical numbers with any length of numbers. The distributive law was not as beneficial to the development of the general model as summation of corresponding lagged multiplicand along with multiplicative identity whereas it was invaluable when combined with identity and commutative laws in formulating specific models for digits two (2) to nine (9). Bearing in mind the laborious task involved in multiplication process of any long length of numbers with identical numbers, an algorithm was developed with JAVA programming language for MIN model to enhance better performance than calculator or Excel package. Results of MIN model were displayed completely without approximation involving exponent compared to results of calculator or Excel package. Therefore MIN model is found to be preferred.

Keywords: *Multiplication; Summation; Lag; Identical numbers, MIN Model.*

1. Introduction

Multiplication is one of the mathematical operations in elementary arithmetic (others being addition, subtraction and division) often denoted by cross symbol "x" of scaling one number or variable by another (Fine, 1907; Merzbach, 1991). This cross symbol "x" introduced by William Oughtred in seventeenth century also known as St. Andrew's cross or dimension sign or into sign (Cajori, 1919; Stallings, 2000) while the middle dot (.) is commonly connotated with more advanced or scientific use for multiplication internationally (Browell, 1939; Julius, 2013).

The multiplication of any number or variable and the identity value give the same number or variable as the result. The number "one" (1) is the identity property of multiplication called multiplicative identity (Peano axioms) (White head and Russel, 1913). Other properties of multiplication include commutative, associative and distributive.

The ability to multiply any number or variable with one (1) and still retains its identity prompted this study in relation to identical numbers in multiplication processes and the formulation of a model for the procedures.

2. Multiplication of Identical Numbers

One (1) as a multiplicative identity is an element of natural numbers (N) otherwise called counting numbers. However, one (1) may be replicated to form other elements of the natural numbers such as 11, 111, 1111, … that are identical in nature (property wise) and other elements of the natural numbers can be replicated to form identical numbers include digits two (2) to nine (9).

Cutler and Trachtenberg (1960), Benjamin and Shermer (2006), Sawyer (2006) extended multiplicative identity of one (1) on eleven (11) to multiply by any length of numbers (multiplicand) starting with the first and last digits (they remain the same except where carrying of number is applicable to be added to the next first digit) inserting the sums of adjacent pairs of digits sequentially in between (Appendices 1 and 2).

Further study of distributive property of multiplication [a(b+c) = ab + ac] into the study of Cutler and Trachtenberg (1960) decomposed eleven (11) as ten (10) plus one (1) (that is, 11 = 10 + 1) reflecting the multiplication of any length of numbers by ten (10) and one (1) as transferring of zero (0) to the number multiplied by ten (10) while multiplicative identity for one (1) still holds and the resulting summation remains unchanged when compared with multiplication of eleven (11) (Jason, 2010) (Appendix 3).

The combination of identity, commutative and distributive properties of multiplication in determining the results for multiplication of identical numbers as to relating digits one (1) to nine (9) will be useful in achieving an understandable model for this study.

Multiplication of any length of numbers by identical numbers of digit one (1) reflected distributive and identity properties of multiplication (Appendix 4) (Jason, 2010) while multiplicative identity extended for eleven (11) only (Cutler and Trachtenberg, 1960; Benjamin and Shermer, 2006; Sawyer, 2006). However, multiplication of identical numbers of digits two (2) to nine (9) involves identity, commutative and distributive laws of multiplication (Appendix5). This enhances effective results for multiplication of identical numbers between digits one (1) to nine (9) which can be extended to any length of numbers as replicated independent digits of one (1) to nine (9) are stochastically identical (Table 1).

3. General Presentation of the MIN Model

The MIN model is to ease the processes of multiplication involving identical numbers from digits one (1) to nine (9). It is therefore necessary to have first a pertinent and stable specification of the relationships, which are very often largely summation of lagged multiplicands. See Appendix 6

Typically, the general MIN model $\sum_{j=0}^{n} X_{i-j}$ (where random variable X is the multiplicand, X_i is the digits of the multiplicand and X_{i-j}; j = 0,1,2,... n are the lagged multiplicands) is build around the premises of multiplicative identity. However, further study into the general MIN model involving commutative and distributive properties of multiplication lead to some specific models for multiplication of identical numbers of digits two (2) to nine (9) (Appendix 7).

In furtherance of the MIN model, an algorithm was developed using JAVA programming language (version JDK 1.7, in a NETBEANS Integrated Development Environment, IDE) as to enabling competion with calculator and Excel package (Appendices 8 and 9) in achieving quick, faster and accurate results in terms of time management and calculations without approximation or exponent.

4. Discussion and Conclusion

The present study demonstrates that Multiplication of Identical Numbers (MIN) is enhanced with summation along identity, commutative, associative and distributive properties of multiplication (Sawyer, 2006 and Jason, 2010). It also identifies any length of numbers (multiplicand) multiply by identical numbers as a variable and the digits (values) of the multiplicand as its respective variates.

In this study, a better and faster method is formulated as MIN model for multiplication of identical numbers of any length of digits. On the basis of this investigation, the MIN model should constitute a reliable method for calculating the multiplication of identical numbers as it will assist students with weak background in multiplication concepts and results are explicitly stated without any approximation when large numbers are involved unlike calculator and Excel package.

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Table 1

Identical Numbers formed from digits one (1) to nine (9)

Digit	Identical Numbers						
1	11	111	1111	11111	111111	1111111	...
2	22	222	2222	22222	222222	2222222	...
3	33	333	3333	33333	333333	3333333	...
4	44	444	4444	44444	444444	4444444	...
5	55	555	5555	55555	555555	5555555	...
6	66	666	6666	66666	666666	6666666	...
7	77	777	7777	77777	777777	7777777	...
8	88	888	8888	88888	888888	8888888	...
9	99	999	9999	99999	999999	9999999	...

Appendix 1

Multiplication of any length of numbers by eleven (11) for carrying of digits not applicable

Examples

1. Multiply 23 by 11

   2 (2 + 3) 3

   2 5 3

   23 x 11 = 253

2. Multiply 234 by 11

   2 (2+3) (3 + 4) 4

   2 5 7 4

   234 x 11 = 2574

3. Multiply 7142 by 11

   7 (7 +1) (1 + 4) (4 + 2) 2

   7 8 5 6 2

   7142 x 11 = 78562

4. Multiply 27145 by 11

   2 (2 + 7) (7 + 1) (1 + 4) (4 + 5) 5

   2 9 8 5 9 5

   27145 x 11 = 298595

5. Multiply 222222 by 11

   2 (2 + 2) (2 + 2) (2 + 2) (2 + 2) (2 + 2) 2

   2 4 4 4 4 4 2

   222222 x 11 = 2444442

Appendix 2

Multiplication of any length of numbers by eleven (11) for carrying of digits applicable

Examples

1. Multiply 79 by 11

```
7 (7 + 9) 9
7 (16) 9
(7 + 1) 6     9
----------------
8         6   9
----------------
79 x 11 = 869
```

2. Multiply 877 by 11

```
8 (8 + 7) (7 + 7) 7
8 (15)     (14)   7
8  (15 + 1)   4   7
8     (16)    4   7
(8+1)    6    4   7
9        6    4   7
--------------------
877 x 11   = 9647
```

3. Multiply 9787 by 11

```
9 (9+7)  (7+8) (8+7)  7
9  (16)  (15)    (15) 7
9  (16)  (15+1)    5  7
9  (16+1)    6     5  7
9    (17)    6     5  7
(9+1)   7    6     5  7
(10)    7    6     5  7
1   0   7    6     5  7
9787  x  11    =    107657
```

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Appendix 3

Multiplication of any length of numbers by eleven (11) using distributive and identity properties.

Examples

1. Multiply 79 by 11

79 x 11
79 x (10 + 1)
(79 x 10) + (79 x 1)
790 + 79

	7	9	0
+		7	9
	8	6	9

79 x 11 = 869

2. Multiply 888 by 11

888 x 11
888 x (10 + 1)
(888 x 10) + (888 x 1)
8880 + 888

	8	8	8	0
+		8	8	8
	9	7	6	8

888 x 11 = 9768

3. Multiply 9787 by 11

9787 x 11
9787 x (10 + 1)
(9787 x 10) + (9787 x 1)
97870 + 9787

	9	7	8	7	0
+	9	7	8	7	
1	0	7	6	5	7

9787 X 11 = 107657

Appendix 4

Multiplication of any length of numbers by identical numbers of digits one (1)* using distributive and identity properties.

Examples

1. Multiply 35 by 111
35 x 111
35 x (100 + 10 + 1)
(35 x 100) + (35 x 10) + (35 x 1)
3500 + 350 + 35

	3	5	0	0
+		3	5	0
			3	5
	3	8	8	5

35 x 111 = 3885

2. Multiply 38725 by 1111
38725 x 1111
38725 x (1000 + 100 + 10 + 1)
(38725 x 1000) + (38725 x 100) + (38725 x 10) + (38725 x 1)
38725000
3872500
\+ 387250
38725
43023475
38725 x 1111 = 43023475

3. Multiply 875 x 111111
875 x 111111
875 x (100000 + 10000 + 1000 + 100 + 10 + 1)
87500000 + 8750000 + 875000 + 87500 + 8750 + 875 = 97222125
875 x 111111 = 97222125

*Multiplication of any length of numbers by eleven (11) is in appendix 3 which is also an identity number of digit one (1).

Appendix 5

Multiplication of any length of numbers by identical numbers of digits two (2) to nine (9) using commutative, distributive and identity properties.

Example

Multiply 653 by 999
653 x 999
653 x 9 x 111
(653 x 111) x 9
(653 x 111) x (4 + 4 + 1)
(653 x 111) x [(2+2) + 4 + 1]
72483 x [(2+2) + 4 + 1]
[72483 x (2+2)] + (72483 x 4) + (72483 x 1)
[(72483 x 2) + (72483 x 2)] + (72483 x 4) + (72483 x 1)
2(72483 x 2) + (72483 x 4) + (72483 x 1)

		7	2	4	8	3
+		7	2	4	8	3
	1	4	4	9	6	6

72483 x 2 = 144966
2(144966) + (72483 x 4) + (72483 x 1)
(144966 x 2) + (72483 x 4) + (72483 x 1)

	1	4	4	9	6	6
+	1	4	4	9	6	6
	2	8	9	9	3	2

144966 x 2 = 289932 = 72483 x 4
(289932 + 289932) + (72483 x 1)
2(289932) + (72483 x 1)
(289932 x 2) + (72483 x 1)

	2	8	9	9	3	2
+	2	8	9	9	3	2
	5	7	9	8	6	4

289932 x 2 = 579864
579864 + 72483

	5	7	9	8	6	4
+		7	2	4	8	3
	6	5	2	3	4	7

653 x 999 = 652347

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Appendix 6

Generalization of the MIN model

Examples

1. Multiply 6537 by 11

   6537 is the multiplicand (X_i)

   $\underset{1\,0}{11}\, is X_{1-j};\ j = 0,1$

   $$\sum_{j=0}^{1} X_{i-j} = X_1 + X_{i-1}$$

X_i	6	5	3	7	
X_{i-1}		6	5	3	7
$\sum_{j=0}^{1} X_{i-j}$	7	1	9	0	7

6537 x 11 = 71907

2. Multiply 79856576 by 111111

   79856576 is the multiplicand (X_i)

   $\underset{5\,4\,3\,2\,1\,0}{111111}$ is X_{i-j} ; j = 0, 1, 2, 3, 4, 5

   $$\sum_{j=0}^{5} X_{i-j} = X_i + X_{i-1} + X_{i-2} + X_{i-3} + X_{i-4} + X_{i-5}$$

X_i	7	9	8	5	6	5	7	6					
X_{i-1}		7	9	8	5	6	5	7	6				
X_{i-2}			7	9	8	5	6	5	7	6			
X_{i-3}				7	9	8	5	6	5	7	6		
X_{i-4}					7	9	8	5	6	5	7	6	
X_{i-5}						7	9	8	5	6	5	7	6
$\sum_{j=0}^{5} X_{i-j}$	8	8	7	2	9	4	4	0	1	5	9	3	6

79856576 x 111111 = 8.872944016 x 10^{12} (Calculator)

79856576 * 111111 = 8.8729 E12 (Excel Package)

The general MIN model is $\sum_{j=0}^{n} X_{i-j}$

Appendix 7

Formulation of Specific MIN Model

Digit	Model	Example
1.	$\sum_{j=0}^{n} X_{i-j}$	6537 x 11 = 71907
2.	$\sum_{j=0}^{n} X_{i-j} + \sum_{j=0}^{n} X_{i-j} = 2\sum_{j=0}^{n} X_{i-j}$	6537x22=2(6537x11)=(6537x11)+(6537x11)=143814
3.	$2\sum_{j=0}^{n} X_{i-j} + \sum_{j=0}^{n} X_{i-j} = 3\sum_{j=0}^{n} X_{i-j}$	6537x33=3(6537x11)=2(6537x11)+(6537x11)=2157521
4.	$2\sum_{j=0}^{n} X_{i-j} + 2\sum_{j=0}^{n} X_{i-j} = 4\sum_{j=0}^{n} X_{i-j}$	6537x44=4(6537x11)=2(6537x11)+2(6537x11)=287628
5.	$4\sum_{j=0}^{n} X_{i-j} + \sum_{j=0}^{n} X_{i-j} = 5\sum_{j=0}^{n} X_{i-j}$	6537x55=5(6537x11)=4(6537x11)+(6537x11)=359535
6.	$4\sum_{j=0}^{n} X_{i-j} + 2\sum_{j=0}^{n} X_{i-j} = 6\sum_{j=0}^{n} X_{i-j}$	6537x66=6(6537x11)=4(6537x11)+(6537x11)=431442
7.	$4\sum_{j=0}^{n} X_{i-j} + 3\sum_{j=0}^{n} X_{i-j} = 7\sum_{j=0}^{n} X_{i-j}$	6537x77=7(6537x11)=4(6537x11)+3(6537x11)=503349
8.	$4\sum_{j=0}^{n} X_{i-j} + 4\sum_{j=0}^{n} X_{i-j} = 8\sum_{j=0}^{n} X_{i-j}$	6537x88=8(6537x11)=4(6537x11)+4(6537x11)=575256
9.	$4\sum_{j=0}^{n} X_{i-j} + 5\sum_{j=0}^{n} X_{i-j} = 9\sum_{j=0}^{n} X_{i-j}$	6537x99=9(6537x11)=4(6537x11)+5(6537x11)=647163