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A COMMON FIXED POINT THEOREM FOR FOUR SELF MAPS ON AN INTUITIONISTIC FUZZY METRIC SPACE

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1. INTRODUCTION

Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [7] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm. Recently, in 2006, Alaca et al. [2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space due to Kramosil and Michalek [5]. In 2006, Turkoglu et al. [9] proved Jungck's common fixed point theorem [4] in the setting of intuitionistic fuzzy metric spaces for commuting mappings. For more detailS, one can refer to papers ([1], [6], [10], [11]).

In this paper, we prove a common fixed point theorem for four self mappings satisfying contractive condition in intuitionistic fuzzy metric spaces. We observe that the result of Manro [12] follows as a corollary.

2. PRELIMINARIES

Definition 2.1(Zadeh, [13]): Fuzzy set A in X is a function with domain X and values in [0, 1]

Schweizer and Sklar [8] defined the following notion:

Definition 2.2 A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if * satisfies the following conditions:

(i) * is commutative and associative
(ii) * is continuous
(iii) $a * 1 = a$ for all $a \in [0,1]$
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

Definition 2.3 A binary operation $\diamond: [0,1] \times [0,1] \to [0,1]$ is a continuous t-conorm if $\diamond$ satisfies the following conditions

(i) $\diamond$ is commutative and associative
(ii) $\diamond$ is continuous

(iii) a ⋄ 0 = a for all a ϵ [0,1]

(iv) a ⋄ b ≤ c ⋄ d whenever a ≤ c and b ≤ d for all a, b, c, d ϵ [0,1]

Alaca et al. [2] defined the notion of a intuitionistic fuzzy metric space as follows:

Definition 2.4 A 5-tuple (X, M, N,*,⋄) is said to be an intuitionistic fuzzy metric space if X is a non empty set, * is a continuous t-norm, ⋄ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying

(i) $M(x,y,t) + N(x,y,t) \leq 1 \ \forall\, x,y \in X \ and \ t > 0;$

(ii) $M(x,y,0) = 0 \ \forall\, x,y \in X$

(iii) $M(x,y,t) = 1 \ \forall\, x,y \in X \ and \ t > 0 \ if \ and \ only \ if \ x = y;$

(iv) $M(x,y,t) = M(y,x,t) \ \forall\, x,y \in X \ and \ t > o;$

(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s) \ \forall\, x,y \in X \ and \ s,t > 0$

(vi) $\forall\, x,y \in X, M(x,y,\ \cdot): [0,\infty) \rightarrow [0,1] is \ left \ continuous;$

(vii) $\lim_{t\to\infty} M(x,y,t) = 1 \ \forall\, x,y \in X$

(viii) $N(x,y,0) = 1 \ \forall\, x,y \in X$

(ix) $N(x,y,t) = 0 \ \forall\, x,y \in X$and t > 0 if and only if $x = y$

(x) $N(x,y,t) = N(y,x,t) \ \forall\, x,y \in X \ and \ t > 0;$

(xi) $N(x,y,t) \ \diamond \ N(y,z,s) \geq N(x,z,t+s) \ \forall\, x,y \in X \ and \ s,t > 0$

(xii) $\forall\, x,y \in X, N(x,y,\ \cdot): [0,\infty) \rightarrow [0,1] is \ right \ continuous;$

(xiii) $\lim_{t\to\infty} N(x,y,t) = 0 \ \forall\, x,y \in X.$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non nearness between x and y with respect to t, respectively.

Remark 2.5 Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *,⋄) such that t-norm * and t-conorm ⋄ are associated as x ⋄ y = 1-((1-x) * (1-y)) $\forall\, x,y \in X$

Remark 2.6 In an intuitionistic fuzzy metric space (X, M, N, *,⋄), M(x, y,·) is non-decreasing and N(x, y, ·) is non-increasing $\forall\, x,y \in X$.

Definition 2.7 Let (X, M, N, *,⋄) be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be (i) convergent to a point x ∈ X if

$$\lim_{n\to\infty} M(x_n,x,t) = 1 \ and \ \lim_{n\to\infty} N(x_n,x,t) = 0 \ \forall\, t > 0$$

(ii) Cauchy sequence if

$$\lim_{n\to\infty} M\left(x_{n+p},x_n,t\right) = 1 \ and \ \lim_{n\to\infty} N\left(x_{n+p},x_n,t\right) = 0 \ \forall\, t > 0, and \ p > 0$$

Definition 2.8 An intuitionistic fuzzy metric space (X, M, N, *,⋄) is said to be complete if and only if every Cauchy sequence in X is convergent.

Turkoglu et al. [10] defined the following notions:

Definition 2.9 Let A and S be self-mappings of an intuitionistic fuzzy metric space (X, M, N, *,⋄). Then the pair (A, S) is said to be commuting if

$$M(ASx, SAx, t) = 1 \text{ and } N(ASx, SAx, t) = 0 \ \forall\, x \in X, t > 0$$

Definition 2.10 Let A and S be self-mappings of an intuitionistic fuzzy metric space (X, M, N, *,⋄). Then the pair (A, S) is said to be weakly commuting if

$M(ASx, SAx, t) \geq M(Ax, Sx, t)$ and $N(ASx, SAx, t) \leq N(Ax, Sx, t)\ \forall\, x \in X, t > 0.$

Definition 2.11 Let A and S be self-mappings of an intuitionistic fuzzy metric space (X, M, N, *,⋄)

Thenthe pair (A, S) is said to be compatible if

$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ and $\lim_{n\to\infty} N(ASx_nSAx_n, t) = 0\ \forall\, t > 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u$ for some $u \in X$

Definition 2.12(D.Turgoklu et.al., [9]) Let A and S be self-mappings of an intuitionistic fuzzy metric space (X, M, N, *, ⋄)

Then the pair (A, S) is said to be point wise R-weakly commuting if given x$\in X$, there exists R>0 such that

$M(ASx, SAx, t) \geq M(Ax, Sx, t/R)$ and $N(ASx, SAx, t) \leq N(Ax, Sx, t/R)\ \forall\, x \in X$ and $t > 0.$

Clearly, every pair of weakly commuting mappings is point wise R-weakly commuting with R = 1.

Definition 2.13(S.Muralisankar, G.Kalpana, [6]) Let A and S be self-mappings of an intuitionistic fuzzy metric space (X, M, N, *,⋄)

Then the pair (A, S) is said to be reciprocally continuous if

$ASu_n \to Az$ and $SAu_n \to Sz$, whenever $\{u_n\}$ is a sequence such that $Au_n \to z$ and $Su_n \to z$, for some z $\in X$ as $n \to \infty$.

If A and S are both continuous, then they are obviously reciprocally continuous, but converse is not true.

Lemma 2.14 ([2], [9]) Let $\{u_n\}$ be a sequence in an intuitionistic fuzzy metric space (X, M, N, *, ⋄). If there exists a constant k $\in (0, 1)$ such that

$$M(u_n, u_{n+1}, kt) \geq M(u_{n-1}, u_n, t) \text{ and } N(u_n, u_{n+1}, kt) \leq N(u_{n-1}, u_n, t)$$

for n = 1, 2, 3…… then $\{u_n\}$ is a Cauchy sequence in X.

Lemma 2.15 [2] [9] Let (X, M, N, *, ⋄) be an intuitionistic fuzzy metric space. If there exists a constant k ∈ $(0, 1)$ such that

$M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ for all x, y ∈ X and $t > 0$, then x = y.

Now we state the Lemma and Theorem of Manro [12]

Lemma 2.16 [12] Let (X, M, N, *,⋄) be a complete intuitionistic fuzzy metric space with t * t ≥ t and (1-t) $\diamond (1-t) \leq (1-t) \forall t \in [0,1]$. Further, let A, B, S and T be four self mappings of X satisfying

(C1) A(X)⊂ T(X) and B(X) ⊂ S(X),

(C2) there exists a constant k ∈ $(0, 1)$ such that

$$\begin{gathered}[1 + aM(Sx, Ty, kt)] * M(Ax, By, kt) \\ \geq a[M(Ax, Sx, kt) * M(By, Ty, kt) * M(By, Sx, kt)] + M(Ty, Sx, t) * M(Ax, Sx, t) * M(By, Ty, t) \\ * M(By, Sx, \propto t) * M(Ax, Ty, (2-\propto)t)\end{gathered}$$

and

$$\begin{gathered}[1 + aN(Sx, Ty, kt)] \diamond N(Ax, By, kt) \\ \leq a[N(Ax, Sx, kt) \diamond N(By, Ty, kt) \diamond N(By, Sx, kt)] + N(Ty, Sx, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \\ \diamond N(By, Sx, \propto t) \diamond N(Ax, Ty, (2-\propto)t)\end{gathered}$$

for all x, y ∈ $X, a \geq 0, \propto \in (0,2)$ and $t > 0$.

If the pairs (A, S) and (B, T) are point wise R-weakly commuting then continuity of one of the mappings in compatible pair (A, S) or (B, T) implies their reciprocal continuity.

Theorem 2.1 [12] Let (X, M, N, *,⋄) be a complete intuitionistic fuzzy metric space with t * t ≥ t and (1-t)⋄ $(1-t) \leq (1-t) \forall t \in [0,1]$. Further, let A, B, S and T be four self mappings of X satisfying (C1) and (C2). If the pairs (A,S) and (B,T) are compatible and point wise R-weakly commuting and one of the mappings in compatible pair (A,S) or (B,T) is continuous, then A,B,S and T have a unique common fixed point.

3. MAIN RESULTS

Lemma 3.1 Let (X, M, N, *,⋄) be a complete intuitionistic fuzzy metric space with t * t ≥ t and (1-t) $\diamond (1-t) \leq (1-t) \; \forall t \in [0,1]$. Further, let A, B, S and T be four self mappings of X satisfying

(C1) A(X) ⊂ T(X) and B(X) ⊂ S(X),

(C2)' there exist constants k ∈ $(0,1)$ and $q \in (0,1)$ such that $0 < k < q$

$$M(Ax, By, kt) \geq M(Ty, Sx, t) * M(Ax, Sx, t) * M(By, Ty, t)$$

$$* M(By, Sx, (1+q)t) * M(Ax, Ty, qt); q \in (0,1)$$

and

$$N(Ax, By, kt) \leq N(Ty, Sx, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t)$$

$$\diamond N(By, Sx, (1+q)t) \diamond N(Ax, Ty, qt)$$

for all x, y $\in X$ and $t > 0$.

If the pairs (A, S) and (B, T) are compatible then continuity of one of the mappings in compatible pair (A, S) or (B, T) implies their reciprocal continuity.

Proof: First, assume that A and S are compatible and S is continuous. We show that A and S are reciprocally continuous. Let $\{u_n\}$ be a sequence such that $Au_n \to z$ and $Su_n \to z$, for $z \in X$ as $n \to \infty$.

Since S is continuous, we have $SAu_n \to Sz$ and $SSu_n \to Sz$, for $z \in X$ as $n \to \infty$ and since (A, S) is compatible, we have

$$\lim_{n\to\infty} M(ASu_n, SAu_n, t) = 1$$

This implies that

$$\lim_{n\to\infty} M(ASu_n, SAz, t) = 1 \ \forall t > 0$$

That is $ASu_n \to Sz$ as $n \to \infty$. By (C1) for each n, there exists $u_n \in X$ such that $ASu_n = Tv_n$. Thus we have $SSu_n \to Sz, SAu_n \to Sz, ASu_n \to Sz$ and $Tv_n \to Sz$ as $n \to \infty$, whenever $ASu_n = Tv_n$.

Now we claim that $Bv_n \to Sz$ as $n \to \infty$. In (C2)[1] put x = Su_n and y = v_n

$$M(ASu_n, Bv_n, kt) \geq M(Tv_n, SSu_n, t) * M(ASu_n, SSu_n, t) * M(Bv_n, Tv_n, t)$$
$$* M(Bv_n, SSu_n, (1+q)t) * M(ASu_n, Tv_n, qt); q \in (0,1)$$

and

$$N(ASu_n, Bv_n, kt) \leq N(Tv_n, SSu_n, t) \diamond N(ASu_n, SSu_n, t) \diamond N(Bv_n, Tv_n, t)$$
$$\diamond N(Bv_n, SSu_n, (1+q)t) \diamond N(ASu_n, Tv_n, qt)$$

$$M(Sz, Bv_n, kt) \geq M(Sz, Sz, t) * M(Sz, Sz, t) * M(Bv_n, Sz, t)$$
$$* M(Bv_n, Sz, (1+q)t) * M(Sz, Sz, qt); q \in (0,1)$$

and

$$N(Sz, Bv_n, kt) \leq N(Sz, Sz, t) \diamond N(Sz, Sz, t) \diamond N(Bv_n, Sz, t)$$
$$\diamond N(Bv_n, Sz, (1+q)t) \diamond N(Sz, Sz, qt); q \in (0,1)$$

$M(Sz, Bv_n, kt) \geq M(Bv_n, Sz, t)$ *and* $N(Sz, Bv_n, kt) \leq N(Bv_n, Sz, t)$

$M(Sz, Bv_n, kt) \to 1$ *and* $N(Sz, Bv_n, kt) \to 0$

Therefore $Bv_n \to Sz$. *Also* $ASu_n = Tv_n$ *implies* $Tv_n \to Sz$

Again by (C2)[I] substitute $Su_n = z$, we get

$$M(Az, Bv_n, kt) \geq M(ASu_n, Sz, t) * M(Az, Sz, t) * M(Bv_n, ASu_n, t) * M(Bv_n, Sz, (1+q)t) * M(Az, ASu_n, qt); q \in (0,1)$$

and

$$N(Az, Bv_n, kt) \leq N(ASu_n, Sz, t) \diamond N(Az, Sz, t) \diamond N(Bv_n, ASu_n, t) \diamond N(Bv_n, Sz, (1+q)t) \diamond N(Az, ASu_n, qt)$$

$$M(Az, Bv_n, kt) \geq M(Az, Sz, t) * M(Az, Sz, t) * M(Bv_n, Az, t) * M(Bv_n, Sz, (1+q)t) * M(Az, Az, qt)$$

and

$$N(Az, Bv_n, kt) \leq N(Az, Sz, t) \diamond N(Az, Sz, t) \diamond N(Bv_n, Az, t) \diamond N(Bv_n, Sz, (1+q)t) \diamond N(Az, Az, qt)$$

$$M(Az, Bv_n, kt) \geq M(Az, Sz, t) * (Bv_n, Az, t) \text{ and } N(Az, Bv_n, kt) \leq N(Az, Sz, t) \diamond N(Bv_n, Az, t)$$

$$M(Az, Bv_n, kt) \geq M(Az, Sz, t) \text{ and } N(Az, Bv_n, kt) \leq N(Az, Sz, t)$$

$$By\ Lemma\ 2.14, Az = Sz. But\ ASu_n \to Sz = Az, implies\ ASu_n \to Az\ and\ SAu_n \to Sz$$

Hence (A, S) is reciprocally continuous on X.

Similarly, if the pair (B, T) is compatible and T is continuous then the proof is similar. This completes the proof.

Now we prove our main theorem.

Theorem 3.1 Let (X, M, N, *,⋄) be a complete intuitionistic fuzzy metric space with $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)\ \forall t \in [0,1]$. Further, let A, B, S and T be four self mappings of X satisfying (C1) and (C2)[I]. If the pairs (A, S) and (B, T) are compatible and one of the mappings in compatible pairs (A, S) or (B, T) is continuous, then A, B, S and T have a unique common fixed point.

Proof By (C1) since $A(X) \subset T(X)$, for any point $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for this point $x_1 \in X$, we can choose a point x_2 in X such that $Bx_1 = Sx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that for n = 0, 1, 2…..

$$y_{2n} = Ax_{2n} = Tx_{2n+1} \text{ and } y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$$

In (C2)[I] substitute $x = x_{2n+1}$; $y = x_{2n+1}$

$$M(Ax_{2n+2}, Bx_{2n+1,}kt) \geq M(Tx_{2n+1}, Sx_{2n+2}, t) * M(Ax_{2n+2}, SBx_{2n+2,}t) * M(Bx_{2n+1}, Tx_{2n+1,}t) * M(Bx_{2n+1}, Sx_{2n+2,}(1+q)t) * M(Ax_{2n+1}, Tx_{2n+1,}qt)$$

and

$$N(Ax_{2n+2}, Bx_{2n+1,}kt) \leq N(Tx_{2n+1}, Sx_{2n+2}, t) \diamond N(Ax_{2n+2}, SBx_{2n+2,}t) \diamond N(Bx_{2n+1}, Tx_{2n+1,}t) \diamond N(Bx_{2n+1}, Sx_{2n+2,}(1+q)t) \diamond N(Ax_{2n+1}, Tx_{2n+1,}qt)$$

$$M(y_{2n+2}, y_{2n+1,}kt) \geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+2}, y_{2n+1,}t) * M(y_{2n+1}, y_{2n,}t) * M(y_{2n+1}, y_{2n+1}(1+q)t) * M(y_{2n+1}, y_{2n,}qt)$$

and

$$N(y_{2n+2}, y_{2n+1,}kt) \leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+2}, y_{2n+1,}t) \diamond N(y_{2n+1}, y_{2n,}t) \diamond N(y_{2n+1}, y_{2n+1,}(1+q)t) \diamond N(y_{2n+1}, y_{2n,}qt)$$

$$M(y_{2n+2}, y_{2n+1,}kt) \geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+2}, y_{2n+1,}t) * M(y_{2n+1}, y_{2n,}qt) \rightarrow (3.1.1)$$

and

$$N(y_{2n+2}, y_{2n+1,}kt) \leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+2}, y_{2n+1,}t) \diamond N(y_{2n+1}, y_{2n,}qt) \rightarrow (3.1.2)$$

(3.1.1) implies $M(y_{2n+2}, y_{2n+1,}kt) \geq M(y_{2n+1}, y_{2n,}qt)$

So that $M\left(y_{2n+2}, y_{2n+1,}\frac{k}{q}qt\right) \geq M(y_{2n+1}, y_{2n,}qt)$

(3.1.2) implies $N(y_{2n+2}, y_{2n+1,}kt) \leq N(y_{2n+1}, y_{2n,}qt)$

$$N(y_{2n+2}, y_{2n+1,}(k/q)qt) \leq N(y_{2n+1}, y_{2n,}qt)$$

Case-I: Suppose $y_n = y_{n+1}$ for some n

Then $y_0=Ax_0=Tx_1$ and $y_1=Bx_1=Sx_2$ so that $Ax_0 = Tx_1 = Bx_1 = Sx_2$

Now $y_2=Ax_2=Tx_3$; put $x = x_2$; $y = x_1$ in (C2)'

$$M(Ax_2, Bx_1, kt) \geq M(Tx_1, Sx_2, t) * M(Ax_2, Sx_2, t) * M(Bx_1, Tx_1, t) * M(Bx_1, Sx_2, (1+q)t) * M(Ax_2, Tx_1, qt)$$

and

$$N(Ax_2, Bx_1, kt) \leq N(Tx_1, Sx_2, t) \diamond N(Ax_2, Sx_2, t) \diamond N(Bx_1, Tx_1, t) \diamond N(Bx_1, Sx_2, (1+q)t) \diamond N(Ax_2, Tx_1, qt)$$

$$M(y_2, y_1, kt) \geq M(y_0, y_1, t) * M(y_2, y_1, t) * M(y_1, y_0, t) * M(y_1, y_1, (1+q)t) * M(y_2, y_1, qt)$$

and

$N(y_2, y_1, kt) \leq N(y_0, y_1, t) \diamond N(y_2, y_1, t) \diamond N(y_1, y_0, t) \diamond N(y_1, y_1, (1+q)t) \diamond N(y_2, y_1, qt);$

$$M(y_2, y_1, kt) \geq M(y_2, y_1, t) * M(y_2, y_1, qt) \text{ and } N(y_2, y_1, kt) \leq N(y_2, y_1, t) \diamond N(y_2, y_1, qt)$$

$$M(y_2, y_1, kt) \geq M(y_2, y_1, qt) \text{ and } N(y_2, y_1, kt) \leq N(y_2, y_1, t)$$

Therefore $y_2 = y_1$, and so on $y_3 = y_2$; ……… i.e the sequence converges to y_0

Since (A,S) are compatible $ASx_2=SAx_2 \Rightarrow ABx_1 = SBx_1$ *since* $Sx_2 = Bx_1$ *and* $Ax_2 = Bx_1$

Put x= Bx_1, y = x_1 in (C2)^{l}

$$M(ABx_1, Bx_1, kt) \geq M(Tx_1, SBx_1, t) * M(ABx_1, SBx_1, t) * M(Bx_1, Tx_1, t) * M(Bx_1, SBx_1, (1+q)t) * M(ABx_1, Tx_1, qt)$$

and

$$(ABx_1, Bx_1, kt) \leq N(Tx_1, SBx_1, t) \diamond N(ABx_1, SBx_1, t) \diamond N(Bx_1, Tx_1, t) \diamond N(Bx_1, SBx_1, (1+q)t) \diamond N(ABx_1, Tx_1, qt)$$

$$M(ABx_1, Bx_1, kt) \geq M(Bx_1, ABx_1, t) * M(ABx_1, ABx_1, t) * M(Bx_1, Bx_1, t) * M(Bx_1, ABx_1, (1+q)t) * M(ABx_1, Bx_1, qt)$$

and

$$N(ABx_1, Bx_1, kt) \leq N(Bx_1, ABx_1, t) \diamond N(ABx_1, ABx_1, t) \diamond N(Bx_1, Bx_1, t) \diamond N(Bx_1, ABx_1, (1+q)t) \diamond N(ABx_1, Bx_1, qt)$$

$M(ABx_1, Bx_1, kt) \geq M(ABx_1, Bx_1, qt)$ *And* $N(ABx_1, Bx_1, kt) \leq N(ABx_1, Bx_1, qt)$ *since* $0 < k < q$

Therefore $ABx_1 = Bx_1$. Hence Bx_1 is fixed point of A.

Since (B, T) are compatible $BBx_1 = TBx_1$ (since $Bx_1 = Tx_1$)

Therefore $SBx_1 = ABx_1 = Bx_1 \Rightarrow Bx_1$ is a fixed point of S. But $Ax_0 = Tx_1 = Bx_1 = Sx_2 = Ax_2$ and $ABx_1 = Bx_1$.

$$\Rightarrow ATx_1 = Tx_1, ASx_2 = Sx_2;\ AAx_2 = Ax_2, AAx_0 = Ax_0$$

Now put y = Bx_1 and x = Bx_1, in (C2)^{l} put x = y, then

$$M(ABx_1, BBx_1, kt) \geq M(TBx_1, SBx_1, t) * M(ABx_1, SBx_1, t) * M(BBx_1T, Bx_1, t) * M(BBx_1, SBx_1, (1+q)t) * M(ABx_1, TBx_1, qt)$$

and

$$N(ABx_1, BBx_1, kt) \leq N(TBx_1, SBx_1, t) \diamond N(ABx_1, SBx_1, t) \diamond N(BBx_1T, Bx_1, t) \diamond N(BBx_1, SBx_1, (1+q)t) \diamond N(ABx_1, TBx_1, qt)$$

$$M(Bx_1, BBx_1, kt) \geq M(TBx_1, SBx_1, t) * M(Bx_1, SBx_1, t) * M(BBx_1T, Bx_1, t) * M(BBx_1, SBx_1, (1+q)t) * M(Bx_1, TBx_1, qt)$$

and

$$N(Bx_1, BBx_1, kt) \leq N(TBx_1, SBx_1, t) \diamond N(Bx_1, SBx_1, t) \diamond N(BBx_1T, Bx_1, t) \diamond N(BBx_1, SBx_1, (1+q)t) \diamond N(Bx_1, TBx_1, qt)$$

$$M(Bx_1, BBx_1, kt) \geq M(Bx_1, TBx_1, qt) \text{ and } N(Bx_1, BBx_1, kt) \leq N(Bx_1, TBx_1, qt)$$

$$M(Bx_1, BBx_1, kt) \geq M(Bx_1, BBx_1, qt) \text{ and } N(Bx_1, BBx_1, kt) \leq N(Bx_1, BBx_1, qt)$$

Therefore $BBx_1 = Bx_1$. Hence Bx_1 is a fixed point of B. Similarly Bx_1 is affixed point of T.

Therefore Bx_1 is a common fixed point for A, B, S and T and we are through.

CASE II Assume that $y_n \neq y_{n+1}\ \forall n. Then$

$$M(y_{m+1}, y_m, kt) \geq M(y_m, y_{m-1}, qt) \text{ and } N(y_{m+1}, y_m, kt) \leq N(y_m, y_{m-1}, qt)$$

By continuing, from Lemma 2.1.3 $\{y_n\}$ is convergent. Let $\{y_n\} \to z$.

Now suppose (A, S) is compatible and S is continuous then by Lemma 3.1 A and S are reciprocally continuous so that $ASx_n \to Az$ and $SAx_n \to Sz$

As (A, S) is compatible, we have

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$$

$$\lim_{n\to\infty} M(Az, Sz, t) = 1 \text{ and } \lim_{n\to\infty} N(Az, Sz, t) = 0$$

Hence Az = Sz. Since A(X) ⊂ T(X) there exists p∈ X such that Az = Tp = Sz

In (C2)' put x = z and y = p

$$M(Az, Bp, kt) \geq M(Tp, Sz, t) * M(Az, Sz, t) * M(Bp, Tp, t) * M(Bp, Sz, (1+q)t) * M(Az, Tp, qt)$$

and

$$N(Az, Bp, kt) \geq N(Tp, Sz, t) \diamond N(Az, Sz, t) \diamond N(Bp, Tp, t) \diamond N(Bp, Sz, (1+q)t) \diamond N(Az, Tp, qt)$$

$$M(Az, Bp, kt) \geq M(Az, Bp, t) \text{ and } N(Az, Bp, kt) \leq N(Az, Bp, t)\ \forall\ t > 0$$

Therefore Az = Bp, thus Az= Bp = Sz = Tp.

In (C2)' put x = Az; y =p

$$M(AAz, Bp, kt) \geq M(Tp, SAz, t) * M(AAz, Sz, t) * M(Bp, Tp, t) * M(Bp, SAz, (1+q)t) * M(AAz, Tp, qt)$$

and $N(AAz, Bp, kt) \geq N(Tp, SAz, t) \diamond N(AAz, SAz, t) \diamond N(Bp, Tp, t) \diamond N(Bp, SAz, (1+q)t) \diamond N(AAz, Tp, qt)$

$$M(AAz, Bp, kt) \geq M(Tp, SAz, t) * M(AAZ, Bp, qt)$$

$$N(AAz, Bp, kt) \leq N(Tp, SAz, t) \diamond N(AAZ, Bp, qt)$$

$$M(AAz, Bp, kt) \geq M(AAZ, Bp, qt) \text{ and } N(AAz, Bp, kt) \leq N(Bp, AAz, t)$$

Then AAz = Bp = Az implies AAz = Az =SAz. Therefore Az is a common fixed point for A and S

Similarly by (C2)' we get that Az is a common fixed point of B and T. Hence Az is a common fixed point of A, B, S and T.

Uniqueness: Let x and y be two fixed points of A, B, S and T. Then by (C2)'

$M(Ax, By, kt) \geq M(Ty, Sx, t)$ & $N(Ax, By, kt) \leq N(Sx, Ty, qt)$

Therefore x = y.

Now we show that the result of Manro [12] follows as a corollary of our theorem

Corollary 3.1: If (C2) in Theorem 2.1 is replaced by (C2)' in theorem 3.1, then A, B, S and T have unique fixed point.

Proof: Since (C2) implies (C2)' the result follows.

Note: It may be noted that the pointwise R-weak commutativity of the pairs (A, S) and (B, T) imposed in theorem 2.1 is successfully avoided.

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