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ESTIMATION OF MEAN TIME TO RECRUITMENT FOR A TWO GRADE MANPOWER SYSTEM WITH INTER-EXIT TIMES AS AN ORDINARY RENEWAL PROCESS USING A DIFFERENT RECRUITMENT POLICY WHEN THRESHOLD HAS TWO COMPONENTS

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Abstract

In this paper**,** for a two grade manpower system, three mathematical models are constructed and using a different univariate policy of recruitment based on shock model approach, the expected time to recruitment is obtained when (i) the loss of manpower due to attrition form a sequence of independent and identically distributed exponential random variables (ii) the exponentialinter – exit times form an ordinary renewal processand (iii) the threshold for each grade has two components. A different probabilistic analysis is used to derive the analytical result.

Keywords : Two grade manpower system, attrition, exit times,ordinary renewal process,univariate policy of recruitment and mean time to recruitment.

AMS Mathematics Subject Classification (2010):990B70, 91B40, 91D35, 60H30

Introduction

In any organization, depletion of manpower is quite common whenever policy decisions are announced. Frequent recruitment to compensate this depletion is costlier and hence a suitable policy decision on recruitment has to be designed.Anunivariate recruitment policy, usually known as MAX policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: ***Recruitment is made whenever the maximum loss of man hours exceeds a breakdown threshold.*** *Many* researchers have studied several problems in manpower planning under different conditions by using different methods. In [2] and [3] the authors have discussed some manpower planning models using Markovian and renewal theoretic approach. In [12], [13], [14] and [15] the authors have obtained the variance of the time to recruitment for a two grade man power system using MAX policy under different conditions on the loss of man power, breakdown thresholds and inter – policy decision times. In [6], [7], [8] and [9] the authors have studied the problem of time to recruitment with two sources of depletion under different conditions on the inter-policy decisions, inter-transfer decisions when the breakdown threshold for each grade has only one component using CUM policy of recruitment. In [4] and [5] the authors has studied this problem using univariate MAX policy of recruitment. In [1] the authors have determined the mean time to recruitment for a single grade manpower system with policy decisions forming the only one source of depletion when the threshold for the cumulative loss of manpower has three components using CUM policy of recruitment. Recently, in [10] and [11] the authors have extended this work for a two grade man power system according as the inter – policy decision times and inter – transfer decision times form the same or different ordinary renewal processes respectively. The objective of this paper is to derive the mean time to recruitment for a two grade manpower system using univariate MAX policy of recruitment when (i) the inter – exit times(exit times can be either voluntary or involuntary) form an ordinary renewal process and (ii) the breakdown threshold for each grade has two components.

Model Description

Consider an organization consisting of two grades (grade A and grade B) in which exit of personnel takes place voluntarily (due to policy decisions, VRS etc.) or involuntarily (death, retirement etc.). Let X_n be the loss of man power in the organization at the n^{th} exit point,n=1,2,3,… .It is assumed that $\{X_n\}_{n=1}^{\infty}$ is a sequence of independent and identically distributed exponential random variables with tail distribution $\bar{G}(.)$ and mean $\frac{1}{\alpha}$ ($\alpha > 0$). Let $\chi_A(.)$ indicator function of the event A. Let Z_n be the maximum loss of man power in the organization corresponding to the first n exits. Let U_n be the time between $(n-1)^{th}$ and the n^{th} exit times, n=1,2,3,… and R_{i+1} be the waiting time upto $(i+1)^{th}$ exit. It is assumed that $\{U_n\}_{n=1}^{\infty}$ is an ordinary renewal process with mean $\frac{1}{\lambda}$ ($\lambda > 0$). Let T be the breakdown threshold for the maximum loss of man power in the organization with probability density function h(.) and laplace transform $\bar{h}(.)$. For grade A, let T_{A1} be the normal exponential threshold of depletion of manpower with mean $\frac{1}{\theta_{A1}}$ ($\theta_{A1} > 0$), T_{A2} be the exponential threshold of frequent breaks of existing workers with mean $\frac{1}{\theta_{A2}}$ ($\theta_{A2} > 0$). For grade B, let T_{B1}, T_{B2} be the normal exponential threshold of depletion of manpower, exponential threshold of frequent breaks of existing workers with means $\frac{1}{\theta_{B1}}, \frac{1}{\theta_{B2}}$ ($\theta_{B1},\theta_{B2} > 0$) respectively. The following recruitment policy, known as MAX policy of recruitment is employed in the present work. ***Recruitment is done whenever the maximum loss of man power in the organization exceeds the threshold level T.*** Let W be the time to recruitment with mean E(W).

Main Results

By the recruitment policy, recruitment is done whenever the maximum loss of man power exceeds the threshold T. When the first exit takes place, recruitment would not have been done for U_1 units of time. If the loss of man power X_1 ($=Z_1$) at the first exit point is greater than T, then recruitment is done and in this case $W = U_1 = R_1$. However, if $Z_1 \leq T$ the non–recruitment period will continue till the next exit point. If the cumulative sum Z_2 of the loss of manpower upto the second exit point exceeds T, then recruitment is done and $W = U_1 + U_2 = R_2$. If $Z_2 \leq T$, then the non – recruitment period will continue till the next exit point and depending on $Z_3>T$ or $Z_3 \leq T$, recruitment is done or the non – recruitment period continues and so on. This observation lead to the following expression for the time to recruitment.

$$W = \sum_{i=0}^{\infty} R_{i+1}\chi(Z_i \leq T < Z_{i+1}) ----------(1)$$

Since $E(R_{i+1}) = \sum_{j=0}^{i} E(U_{j+1})$, from (1) we get

$$E(W) = \sum_{i=0}^{\infty}\sum_{j=0}^{i} E(U_{j+1})P(Z_i \leq T < Z_{i+1}) ---(2)$$

Using the law of total probability and the result $E(U_{k+1})=(k+1)E(U)$, k=0,1,2,… in (2), we get

$$E(W) = E(U)\bar{h}(\alpha)\bar{h}(-2\alpha) -----------(3)$$

We now consider different forms for T and obtain the mean for time to recruitment

Case (i): $T = \max(T_{A1}+T_{A2}, T_{B1}+T_{B2})$

The present case for the breakdown threshold T is the best choice when we permit mobility of personnel from one grade to another in order to compensate loss of man power which is larger among the two grades. In this case it is found that

$$h(t) = \frac{\theta_{B1}\theta_{B2}(e^{-(\theta_{B2})t} - e^{-(\theta_{B1})t})}{(\theta_{B1}-\theta_{B2})} + \frac{\theta_{A1}\theta_{B1}\theta_{B2}(e^{-(\theta_{A2}+\theta_{B1})t} - e^{-(\theta_{A2}+\theta_{B2})t})}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})} + \frac{\theta_{A2}\theta_{B1}\theta_{B2}(e^{-(\theta_{A1}+\theta_{B2})t} - e^{-(\theta_{A1}+\theta_{B1})t})}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})} + \frac{\theta_{A1}\theta_{A2}(e^{-(\theta_{A2})t} - e^{-(\theta_{A1})t})}{(\theta_{A1}-\theta_{A2})} +$$

$$\frac{\theta_{A1}\theta_{A2}\theta_{B1}(e^{-(\theta_{A1}+\theta_{B2})t}-e^{-(\theta_{A2}+\theta_{B2})t})}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}+\frac{\theta_{A1}\theta_{A2}\theta_{B2}(e^{-(\theta_{A2}+\theta_{B1})t}-e^{-(\theta_{A1}+\theta_{B1})t})}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}\text{------}(4)$$

Since Z_i and T are independent, using (4) in (3) and on simplification, we find the following expression for mean time to recruitment for the present case.

$$E(W)=\frac{1}{\lambda}\,\bar{h}(\alpha)\bar{h}(-2\alpha)\text{-------------------}(5)$$

where

$$\bar{h}(\alpha)=\Big\{\alpha\Big[\Big(\frac{\theta_{B1}\theta_{B2}}{(\alpha+\theta_{B1})(\alpha+\theta_{B2})}\Big)+\Big(\frac{\theta_{A1}\theta_{A2}}{(\alpha+\theta_{A1})(\alpha+\theta_{A2})}\Big)-\Big(\frac{\theta_{A1}\theta_{B1}\theta_{B2}}{(\theta_{A1}-\theta_{A2})(\alpha+\theta_{A2}+\theta_{B1})(\alpha+\theta_{A2}+\theta_{B2})}\Big)+\Big(\frac{\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{A1}-\theta_{A2})(\alpha+\theta_{A1}+\theta_{B1})(\alpha+\theta_{A1}+\theta_{B2})}\Big)-\Big(\frac{\theta_{B1}\theta_{A1}\theta_{A2}}{(\theta_{B1}-\theta_{B2})(\alpha+\theta_{A1}+\theta_{B2})(\alpha+\theta_{A2}+\theta_{B2})}\Big)+\Big(\frac{\theta_{B2}\theta_{A1}\theta_{A2}}{(\theta_{B1}-\theta_{B2})(\alpha+\theta_{A2}+\theta_{B1})(\alpha+\theta_{A1}+\theta_{B1})}\Big)\Big]\Big\}$$

$$\bar{h}(-2\alpha)=\Big\{-2\alpha\Big[\Big(\frac{\theta_{B1}\theta_{B2}}{(-2\alpha+\theta_{B1})(-2\alpha+\theta_{B2})}\Big)+\Big(\frac{\theta_{A1}\theta_{A2}}{(-2\alpha+\theta_{A1})(-2\alpha+\theta_{A2})}\Big)-\Big(\frac{\theta_{A1}\theta_{B1}\theta_{B2}}{(\theta_{A1}-\theta_{A2})(-2\alpha+\theta_{A2}+\theta_{B1})(-2\alpha+\theta_{A2}+\theta_{B2})}\Big)+\Big(\frac{\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{A1}-\theta_{A2})(-2\alpha+\theta_{A1}+\theta_{B1})(-2\alpha+\theta_{A1}+\theta_{B2})}\Big)-\Big(\frac{\theta_{B1}\theta_{A1}\theta_{A2}}{(\theta_{B1}-\theta_{B2})(-2\alpha+\theta_{A1}+\theta_{B2})(-2\alpha+\theta_{A2}+\theta_{B2})}\Big)+\Big(\frac{\theta_{B2}\theta_{A1}\theta_{A2}}{(\theta_{B1}-\theta_{B2})(-2\alpha+\theta_{A2}+\theta_{B1})(-2\alpha+\theta_{A1}+\theta_{B1})}\Big)\Big]\Big\}$$

(5) gives the mean time to recruitment for the present case.

Case (ii): $T=\min(T_{A1}+T_{A2},\ T_{B1}+T_{B2})$

The present case for the breakdown threshold T is the best choice when we assume that transfer of personnel between grades is not permitted. In this case it is found that

$$h(t)=\frac{\theta_{A2}\theta_{B1}\theta_{B2}e^{-(\theta_{A1}+\theta_{B1})t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}-\frac{\theta_{A2}\theta_{B1}\theta_{B2}e^{-(\theta_{A1}+\theta_{B2})t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}+\frac{\theta_{A1}\theta_{B1}\theta_{B2}e^{-(\theta_{A2}+\theta_{B2})t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}-\frac{\theta_{A1}\theta_{B1}\theta_{B2}e^{-(\theta_{A2}+\theta_{B1})t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}-\frac{\theta_{A1}\theta_{A2}\theta_{B2}e^{-(\theta_{A2}+\theta_{B1})t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}+\frac{\theta_{A1}\theta_{A2}\theta_{B2}e^{-(\theta_{A1}+\theta_{B1})t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}+\frac{\theta_{A1}\theta_{A2}\theta_{B1}e^{-(\theta_{A2}+\theta_{B2})t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}-\frac{\theta_{A1}\theta_{A2}\theta_{B1}e^{-(\theta_{A1}+\theta_{B2})t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})}---(6)$$

$$E(W)=\frac{1}{\lambda}\,\bar{h}(\alpha)\bar{h}(-2\alpha)\ -------(7)$$

where

$$\bar{h}(\alpha)=\Big\{\alpha\Big[\Big(\frac{\theta_{A1}\theta_{B1}\theta_{B2}}{(\theta_{A1}-\theta_{A2})(\alpha+\theta_{A2}+\theta_{B1})(\alpha+\theta_{A2}+\theta_{B2})}\Big)-\Big(\frac{\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{A1}-\theta_{A2})(\alpha+\theta_{A1}+\theta_{B1})(\alpha+\theta_{A1}+\theta_{B2})}\Big)+\Big(\frac{\theta_{B1}\theta_{A1}\theta_{A2}}{(\theta_{B1}-\theta_{B2})(\alpha+\theta_{A1}+\theta_{B2})(\alpha+\theta_{A2}+\theta_{B2})}\Big)-\Big(\frac{\theta_{B2}\theta_{A1}\theta_{A2}}{(\theta_{B1}-\theta_{B2})(\alpha+\theta_{A2}+\theta_{B1})(\alpha+\theta_{A1}+\theta_{B1})}\Big)\Big]\Big\}$$

$$\bar{h}(-2\alpha)=\Big\{-2\alpha\Big[\Big(\frac{\theta_{A1}\theta_{B1}\theta_{B2}}{(\theta_{A1}-\theta_{A2})(-2\alpha+\theta_{A2}+\theta_{B1})(-2\alpha+\theta_{A2}+\theta_{B2})}\Big)-\Big(\frac{\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{A1}-\theta_{A2})(-2\alpha+\theta_{A1}+\theta_{B1})(-2\alpha+\theta_{A1}+\theta_{B2})}\Big)+\Big(\frac{\theta_{B1}\theta_{A1}\theta_{A2}}{(\theta_{B1}-\theta_{B2})(-2\alpha+\theta_{A1}+\theta_{B2})(-2\alpha+\theta_{A2}+\theta_{B2})}\Big)-\Big(\frac{\theta_{B2}\theta_{A1}\theta_{A2}}{(\theta_{B1}-\theta_{B2})(-2\alpha+\theta_{A2}+\theta_{B1})(-2\alpha+\theta_{A1}+\theta_{B1})}\Big)\Big]\Big\}$$

(7) gives the mean time to recruitment for the present case.

Case (iii): $T=((T_{A1}+T_{A2})+(T_{B1}+T_{B2}))$

The choice for T cited in case(iii) provides a better maximum allowable loss of man power in the entire organization compared to the choices mentioned in cases (i) and (ii).In this case it can be shown that

$$h(t)=\frac{\theta_{A1}\theta_{A2}^{2}\theta_{B2}e^{-\theta_{A2}t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})(\theta_{A2}-\theta_{B1})}-\frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}e^{-\theta_{B1}t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})(\theta_{A2}-\theta_{B1})}+\frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}e^{-\theta_{B1}t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})(\theta_{A1}-\theta_{B1})}-\frac{\theta_{A1}^{2}\theta_{A2}\theta_{B2}e^{-\theta_{A1}t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})(\theta_{A1}-\theta_{B1})}+$$

$$\frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}e^{-\theta_{B2}t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})(\theta_{A2}-\theta_{B2})} - \frac{\theta_{A1}\theta_{A2}^{2}\theta_{B1}e^{-\theta_{A2}t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})(\theta_{A2}-\theta_{B2})} - \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}e^{-\theta_{B2}t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})(\theta_{A1}-\theta_{B2})} + \frac{\theta_{A1}^{2}\theta_{A2}\theta_{B1}e^{-\theta_{A1}t}}{(\theta_{A1}-\theta_{A2})(\theta_{B1}-\theta_{B2})(\theta_{A1}-\theta_{B2})} \text{----------------------(8)}$$

$$E(W) = \frac{1}{\lambda}\,\overline{h}(\alpha)\overline{h}(-2\alpha) \text{------------------------------(9)}$$

Where

$$\overline{h}(\alpha) = \Big\{\alpha\Big[\Big(\frac{\theta_{A1}\theta_{A2}^{2}(\theta_{B1}+\theta_{B2}-\theta_{A2})}{(\theta_{A1}-\theta_{A2})(\theta_{A2}-\theta_{B2})(\theta_{A2}-\theta_{B1})(\alpha+\theta_{A2})}\Big) + \Big(\frac{\theta_{A2}\theta_{A1}^{2}(\theta_{A1}-\theta_{B1}-\theta_{B2})}{(\theta_{A1}-\theta_{A2})(\theta_{A1}-\theta_{B2})(\theta_{A1}-\theta_{B1})(\alpha+\theta_{A1})}\Big) - \Big(\frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{B1}-\theta_{B2})(\theta_{A1}-\theta_{B1})(\theta_{A2}-\theta_{B1})(\alpha+\theta_{B1})}\Big) + \Big(\frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{B1}-\theta_{B2})(\theta_{A2}-\theta_{B2})(\theta_{A1}-\theta_{B2})(\alpha+\theta_{B2})}\Big)\Big]\Big\}$$

$$\overline{h}(-2\alpha) = \Big\{-2\alpha\Big[\Big(\frac{\theta_{A1}\theta_{A2}^{2}(\theta_{B1}+\theta_{B2}-\theta_{A2})}{(\theta_{A1}-\theta_{A2})(\theta_{A2}-\theta_{B2})(\theta_{A2}-\theta_{B1})(-2\alpha+\theta_{A2})}\Big) + \Big(\frac{\theta_{A2}\theta_{A1}^{2}(\theta_{A1}-\theta_{B1}-\theta_{B2})}{(\theta_{A1}-\theta_{A2})(\theta_{A1}-\theta_{B2})(\theta_{A1}-\theta_{B1})(-2\alpha+\theta_{A1})}\Big) - \Big(\frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{B1}-\theta_{B2})(\theta_{A1}-\theta_{B1})(\theta_{A2}-\theta_{B1})(-2\alpha+\theta_{B1})}\Big) + \Big(\frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{B1}-\theta_{B2})(\theta_{A2}-\theta_{B2})(\theta_{A1}-\theta_{B2})(-2\alpha+\theta_{B2})}\Big)\Big]\Big\}$$

(9) gives the mean time to recruitment for the present case.

Note:

1. If the threshold for grade A(grade B) has a third component corresponding to backup or reservation sources, the analytical result for all the three cases can be obtained by convoluting the distribution of this component with that of the two components considered in this paper.
2. From the organization's point of view, case (iii) is more suitable than cases (i) and (ii) as the time to recruitment is elongated compared to cases (i) and (ii).

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