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Information Measures Of Size Biased Generalized Gamma Distribution

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Abstract

Size Biased distributions arise in practice when observations from a sample are recorded with unequal probabilities and these distributions are very helpful in modelling the observations that fall in non-experimental, non-replicated and non-random categories. In this paper some information measures of size biased generalized gamma distribution (SBGGD) has been discussed viz entropy, entropy estimation, Kullback-Leibler discrimination, data transformation, log-likelihood ratio and Akaike and Bayesian information criterion . The entropy of the SBGGD is obtained using power transformation technique. The properties to the special cases of SBGGD for varying values of parameters has also been discussed.

Keywords: Generalized gamma distribution, size biased generalized gamma distribution, logarithmic moment, entropy, Kullback-Leibler discrimination,Akaike criterion, Bayesian critetion. L R statistic.

1. Introduction

The generalized gamma distribution is extremely popular flexible distribution and was introduced by Stacy and Mihran [4] because it includes as special cases several distributions: the weibull distribution, the exponential distribution, the log-normal distribution, the levy distribution, gamma distribution, Rayleigh distribution, the Half normal distribution, the chi-square distribution. Various authors have suggested GGD as a general life-testing model. Harter [6] derived maximum likelihood estimators of generalized gamma distribution. Prentice [13] obtained maximum likelihood estimators for generalized gamma distribution by using the technique of reparametrization. Hwang and Huang [7] obtained new moment estimation of parameters of the generalized gamma distribution using its characterization. Entropy is the measure of uncertainty have been found very much useful in various fields such as statistical inference, operations, research, languages physics, econometrics etc. The entropy of a random variable is defined in terms of its probability distribution and can be shown to be a good measure of randomness or uncertainty. In information theory various researchers applied the concept of measure of divergence and its generalization. A measure of divergence is used as the way to evaluate the distance (divergence) between any two populations or functions was initially developed by Mahalonobis [11]. A concept of divergence of measure based on the idea of information theoretic entropy first introduced in communication theory by Shannon [17]. Kullback and Leibler [8] introduced the concept of discrimination information measure also known as the relative entropy. The relative information is a measure of the distance between two distributions and it arises as an expected logarithm of the likelihood ratio. Maximum entropy derivation of generalized gamma distribution is given in Kapur [9]. Nadarajah and Zografos [12] recently presented the entropy of generalized gamma distribution in the context of flexible families of distributions. Morteza and Ahmadabadi [10] derived entropy estimation of generalized gamma distribution.

Size biased distribution are special case of the more general form known as weighted distribution. Rao [14] & [15] extended the idea of the methods of ascertainment upon estimation of frequencies and introduced the concept of weighted distributions. Fisher [5] identified various sampling situations that can be modelled by what is called as weighted distributions. Suriya & Jan [16] obtained bayes estimation of parameter of size biased generalized gamma distribution using lindley's approach. If a random variable T has distribution f(t;δ) with unknown parameter δ, then the correspomding size-biased distribution is of the form

$$f(t;\delta)=\frac{t^{\beta} f(t;\delta)}{\mu_{\beta}} \quad , \text{ where } \mu_{\beta}=\int t^{\beta} f(t,\delta)dt$$

When β=(1,2), the simple size-biased and area-biased distributions are obtained respectively.

This paper is organised as follows. Section 2 defines size biased generalized gamma distribution by power transformation technique. In section 3, entropy of SBGGD and its subfamilies are obtained. Entropy estimation is discussed in section 4. Section 5 discusses Kullback-Leibler discrimination in SBGG family. In section 6, we define the data transfommation of SBGGD. Log-likelihood ratio statistic, Akaike and Bayesian information criterion are discussed in section 7. Section 8 gives some brief conclusion.

2. Probability density function of Size biased generalized gamma distribution

The probability density function of generalized gamma distribution with parameters τ, λ, α is given in

$$f(y|\alpha,\tau,\lambda)=\frac{\tau}{\lambda\Gamma(\alpha)}\left(\frac{y}{\lambda}\right)^{\alpha\tau-1}e^{-\left(\frac{y}{\lambda}\right)^{\tau}};\tau,\alpha,\lambda>0 \qquad (2.1)$$

Where λ is a scale parameter and α and τ are shape parameters and mean of GGD is given by

$$Mean=\frac{\lambda\Gamma\left(\alpha+\frac{1}{\tau}\right)}{\Gamma(\alpha)}$$

Using (1.1) & (1.2), the probability density function of size biased generalized gamma distribution is

$$f(y|\alpha,\tau,\lambda)=\frac{y\tau}{\Gamma\left(\alpha+\frac{1}{\tau}\right)\lambda^{2}}\left(\frac{y}{\lambda}\right)^{\alpha\tau-1}e^{-\left(\frac{y}{\lambda}\right)^{\tau}};\tau,\alpha,\lambda>0$$

Dadpay [3] showed that an important property of GG family for information analysis is that the family is closed under power transformation. That is,

$$If \qquad X\sim GG(\alpha,\tau,\lambda),$$

then $$Y=X^{s}\sim GG\left(\alpha,\frac{\tau}{s},\lambda^{s}\right),$$

using above properties

The power transformation of the probability density function of size biased generalized gamma distribution is given by expression

$$g\left(y\middle|\alpha,\frac{\tau}{s},\lambda^{s}\right)=\frac{y\tau}{\Gamma\left(\alpha+\frac{1}{\tau}\right)s\lambda^{2s}}\left(\frac{y}{\lambda^{s}}\right)^{\frac{\alpha\tau}{s}-1}e^{-\left(\frac{y}{\lambda^{s}}\right)^{\frac{\tau}{s}}};\tau,\alpha,\lambda>0 \qquad (2.2)$$

3.Entropy of Size Biased Generalized Gamma Distribution.

The entropy for continuous distribution is given by

$$H(f) = -\int_{-\infty}^{+\infty} f(x)\log f(x)dx = E(-\log f(x)) \tag{3.1}$$

$$H(g) = -E(\log f(y|\alpha,\tau,\lambda))$$

By the definition of expectation and using (2.2) and $Y = X^s$, we have

If $X \sim SBGG(\alpha,\tau,\lambda)$ then

$$E(Y) = \int_0^\infty yf(y)dy = \int_0^\infty \frac{y^2\tau}{\Gamma\left(\alpha+\frac{1}{\tau}\right)s\lambda^{2s}}\left(\frac{y}{\lambda^s}\right)^{\frac{\alpha\tau}{s}-1} e^{-\left(\frac{y}{\lambda^s}\right)^{\frac{\tau}{s}}} dy$$

and by putting $\left(\frac{y}{\lambda^s}\right)^{\frac{\tau}{s}} = r$

$$= \frac{\lambda^s}{\Gamma\left(\frac{1}{\tau}+\alpha\right)}\int_0^\infty e^{-r} r^{\frac{2s}{\tau}+\alpha-1} dr$$

$$E(Y) = \frac{\lambda^s\Gamma\left(\alpha+\frac{2s}{\tau}\right)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)} \tag{3.2}$$

$$E(\log Y) = \int_0^\infty \log yf(y)dy$$

$$= \int_0^\infty \log y \frac{y\tau}{\Gamma\left(\alpha + \frac{1}{\tau}\right) s\lambda^{2s}} \left(\frac{y}{\lambda^s}\right)^{\frac{\alpha\tau}{s}-1} e^{-\left(\frac{y}{\lambda^s}\right)^{\frac{\tau}{s}}} dy$$

$$= \frac{1}{\Gamma\left(\alpha + \frac{1}{\tau}\right)} \int_0^\infty \log\left(\lambda^s r^{\frac{s}{\tau}}\right) r^{\frac{s}{\tau}+\alpha-1} e^{-r} dr$$

$$= \frac{s}{\Gamma\left(\alpha + \frac{1}{\tau}\right)} \int_0^\infty \left(\log\lambda + \frac{1}{\tau}\log(r)\right) r^{\frac{s}{\tau}+\alpha-1} e^{-r} dr$$

$$E(\log Y) = \frac{s\Gamma\left(\alpha + \frac{s}{\tau}\right)}{\Gamma\left(\alpha + \frac{1}{\tau}\right)} \log(\lambda) + \frac{s\frac{\partial}{\partial\alpha}\Gamma\left(\alpha + \frac{s}{\tau}\right)'}{\tau\Gamma\left(\alpha + \frac{1}{\tau}\right)} \qquad (3.3)$$

Setting s=τ in (3.2) and (3.3) we get the following equations

$$E(Y) = \frac{\lambda^\tau \Gamma(\alpha+2)}{\Gamma\left(\alpha + \frac{1}{\tau}\right)} \qquad (3.4)$$

$$E(\log Y) = \log(\lambda) + \frac{1}{\tau}\psi\left(\alpha + \frac{1}{\tau}\right) \qquad (3.5)$$

$$where, \psi\left(\alpha + \frac{1}{\tau}\right) = \frac{\partial \log\Gamma\left(\alpha + \frac{1}{\tau}\right)}{\partial\left(\alpha + \frac{1}{\tau}\right)}$$

Using (3.4) & (3.5) in (3.1) we get entropy of SBGGD as

$$H(g) = -E\left\{\log\left(\frac{y\tau}{\Gamma\left(\alpha + \frac{1}{\tau}\right)\lambda^2} \left(\frac{y}{\lambda}\right)^{\alpha\tau-1} e^{-\left(\frac{y}{\lambda}\right)^\tau}\right)\right\}$$

$$= -E\left(-\log\Gamma\left(\alpha+\frac{1}{\tau}\right)-\log\lambda+\log\tau+\alpha\tau\log y-\alpha\tau\log\lambda-\frac{y^\tau}{\lambda^\tau}\right)$$

$$= \log\Gamma\left(\alpha+\frac{1}{\tau}\right)+\log\lambda-\log\tau-\alpha\tau E(\log y)+\alpha\tau\log\lambda+\frac{1}{\lambda^\tau}E\left(y^\tau\right) \qquad (3.6)$$

$$H(g)=\log\Gamma\left(\alpha+\frac{1}{\tau}\right)+\log\lambda-\log\tau-\alpha\psi\left(\alpha+\frac{1}{\tau}\right)+\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)} \qquad (3.7)$$

Entropy of subfamilies of SBGGD

The entropy of subfamilies of SBGGD given in below table

Table 2. Entropy of subfamilies of SBGGD

Size biased Distribution	α	τ	λ	Entropy
Exponential	1	1	λ	$\log\lambda-\psi(2)+\log\Gamma(2)+2$
Gamma	α	1	λ	$\log\Gamma(\alpha+1)+\log\lambda-\alpha(\psi(\alpha+1)-1)+1$
Weibull	1	τ	λ	$\log\Gamma\left(\frac{1}{\tau}+1\right)+\log\lambda-\log\tau-\psi\left(\frac{1}{\tau}+1\right)+\frac{2}{\Gamma\left(\frac{1}{\tau}+1\right)}$
Generalized normal	α	2	λ	$\log\Gamma\left(\alpha+\frac{1}{2}\right)+\log\lambda-\alpha\psi\left(\alpha+\frac{1}{2}\right)+\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{2}\right)}-0.30$
Half normal	0.5	2	$\sqrt{2\sigma^2}$	$\log\sigma-\frac{1}{2}\psi(1)+23.85$
Rayleigh	1	2	$\sqrt{2\sigma^2}$	$\log\Gamma\left(\frac{3}{2}\right)+\log\sigma-\psi\left(\frac{3}{2}\right)+0.85$
Maxwell Boltzmann	$\frac{3}{2}$	2	λ	$\log\Gamma(2)+\log\lambda-\frac{3}{2}\psi(2)+119.7$
Chi	$\frac{k}{2}$	2	λ	$\log\Gamma\left(\frac{k+1}{2}\right)+\log\lambda-\frac{k}{2}\psi\left(\frac{k+1}{2}\right)+(k+1)(k+2)(k+3)-0.30$

4.Entropy estimation of Size Biased Generalized Gamma Distribution.

The probability density function of size biased generalized gamma distribution can be written in the form

$$g(y|\alpha,\tau,\lambda)=\frac{\tau}{\Gamma\left(\alpha+\frac{1}{\tau}\right)\lambda^{\alpha\tau+1}}e^{\alpha\tau\log y-\left(\frac{y}{\lambda}\right)^{\tau}}\ ;y\geq 0,\tau,\alpha,\lambda>0 \qquad (4.1)$$

The likelihood function $L(y_1.......y_n|\alpha,\tau,\lambda)$ of SBGGD is

$$L(y_1.......y_n|\alpha,\tau,\lambda)=\left(\frac{\tau}{\Gamma\left(\alpha+\frac{1}{\tau}\right)\lambda^{\alpha\tau+1}}\right)^n e^{\alpha\tau\sum_{i=1}^{n}\log y-\sum_{i=1}^{n}\left(\frac{y}{\lambda}\right)^{\tau}}\ ;y\geq o,\tau,\alpha,\lambda>0 \qquad (4.2)$$

$$l(\alpha,\tau,\lambda)=L(y_1.......y_n|\alpha,\tau,\lambda)$$
$$=n\left(\log\tau-\log\Gamma\left(\alpha+\frac{1}{\tau}\right)-(\alpha\tau+1)\log\lambda+\alpha\tau\overline{\log y}-\frac{\bar{y}^{\tau}}{\lambda^{\tau}}\right) \qquad (4.3)$$

$$where,\ \overline{\log y}=\frac{\sum_{i=1}^{n}\log y}{n}\ \&\ \bar{y}^{\tau}=\frac{\sum_{i=1}^{n}\bar{y}^{\tau}}{n}$$

Differentiate (4.3) with respect to α and λ and equate to zero, we get (4.4) and (4.5) as respectively

$$\overline{\log y}=\log(\lambda)+\frac{1}{\tau}\psi\left(\alpha+\frac{1}{\tau}\right) \qquad (4.4)$$

$$\bar{y}^{\tau}=\frac{\lambda^{\tau}(\alpha\tau+1)}{\tau} \qquad (4.5)$$

Using (3.2) and (3.3) we get

$$E(\log Y)=\overline{\log y}=\log(\lambda)+\frac{1}{\tau}\psi\left(\alpha+\frac{1}{\tau}\right) \qquad (4.6)$$

$$E(\bar{y}^{\tau})=\bar{y}^{\tau}=\frac{\lambda^{\tau}(\alpha\tau+1)}{\tau} \qquad (4.7)$$

Putting (4.6) and (4.7) in (3.6) we have

$$= -\left(\log\hat{\tau} - \log\Gamma\left(\hat{\alpha} + \frac{1}{\hat{\tau}}\right) - (\hat{\alpha}\hat{\tau}+1)\log\hat{\lambda} + \hat{\alpha}\hat{\tau}\overline{\log y} - \frac{\bar{y}}{\hat{\lambda}^{\hat{\tau}}} \right) \tag{4.8}$$

$$\hat{H}(g) = \frac{-l(\hat{\alpha},\hat{\tau},\hat{\lambda})}{n} \tag{4.9}$$

5. Kullback-Leibler discrimation

The relative entropy, also known as the Kullback-Leibler divergence, between two probability distributions on a random variable is a measure of the distance between them. In case of discrete random variable the Kullback-Leibler divergence between two probability distributions is defined as

$$K(p\|q) = \sum_{i=1}^{n} p_i \log\frac{p_i}{q_i}$$

and in continuous case

$$K(p\|q) = \int p(x)\log\frac{p(x)}{q(x)}dx$$

Let $SBGG_o = SBGG(\alpha_o, \tau_o, \lambda_o)$ be a given $SBGG$ distribution, then the discrimination information function between $SBGG$ and $SBGG_o$ is given by

$$K(SBGG\|SBGG_o) = \int g(y)\log\frac{g(y)}{g_o(y)}dy$$

$$= \int g(y)\log g(y)dy - \int g(y)g_o(y)dy$$

$$= -H(g) + H(g_o) \tag{7.1}$$

$$H(g_o) = -E[\log g_o(y)] = -E\left[\log\left\{\frac{\tau_o}{\Gamma\left(\alpha_o + \frac{1}{\tau_0}\right)\lambda_o^{\alpha_o\tau_o+1}} y^{\alpha_o\tau_o} e^{-\left(\frac{y}{\lambda_o}\right)^{\tau_o}}\right\}\right]$$

$$= -E\left[\log\tau_o - \log\Gamma\left(\alpha_o + \frac{1}{\tau_o}\right) - (\alpha_o\tau_o + 1)\log\lambda_o + \alpha_o\tau_o\log y - \left(\frac{y}{\lambda_o}\right)^{\tau_o}\right]$$

$$= -\log\tau_o + \log\Gamma\left(\alpha_o + \frac{1}{\tau_o}\right) + (\alpha_o\tau_o + 1)\log\lambda_o - \alpha_o\tau_o E(\log y) + \frac{1}{\lambda_o^{\tau_o}} E\left(y^{\tau_o}\right) \qquad (7.2)$$

Using (3.4) & (3.5), the power and logarithmic moments of SBGG distribution can be written as

$$\mu_s(\alpha,\tau,\lambda) = E_{SBGG}\left(Y^s|\alpha,\tau,\lambda\right) = \frac{\lambda^s\Gamma\left(\alpha + \frac{2s}{\tau}\right)}{\Gamma\left(\alpha + \frac{1}{\tau}\right)} \qquad (7.3)$$

$$v_s(\alpha,\tau,\lambda) = E_{SBGG}\left(\log Y^s\right) = \log\lambda^s + \frac{s}{\tau}\psi\left(\alpha + \frac{1}{\tau}\right) \qquad (7.4)$$

Suppose the class of distribution functions are follows as

$$\eta_\varepsilon = \left(P(y|\varepsilon) : Q_p\left(W_l(y)|\varepsilon\right) = \varepsilon_l, l = 0,1,2\right),$$

$\varepsilon = (\varepsilon_0, \varepsilon_1, \varepsilon_2)$, $\varepsilon_0 = W_0(y) = 1$ normalizes the density function and $W_1(y) = y^\tau$

$$\varepsilon_1 = \mu_\tau(\alpha,\tau,\lambda) = E_{SBGG}\left(Y^s|\alpha,\tau,\lambda\right) = \lambda^\tau \frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha + \frac{1}{\tau}\right)}$$

$$W_2(y) = \log y$$

and $\varepsilon_2 = v(\alpha,\tau,\lambda) = E_{SBGG}(\log Y) = \log\lambda + \frac{1}{\tau}\psi\left(\alpha + \frac{1}{\tau}\right)$ is the geometric mean.

Putting (7.3) & (7.4) in (7.2), the expression reduces to

$$H(g_o) = -\log\tau_o + \log\Gamma\left(\alpha_o + \frac{1}{\tau_o}\right) + \alpha_o\tau_o\log\lambda_o + \log\lambda_o - \alpha_o\tau_o\log\lambda - \frac{\alpha_o\tau_o}{\tau}\psi\left(\alpha + \frac{1}{\tau}\right) + \left(\frac{\lambda}{\lambda_o}\right)^{\tau_o} \frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha + \frac{1}{\tau}\right)} \qquad (7.5)$$

Putting (3.7) and (7.5) in (7.1), it gives the following expression

$$K(g\|g_o) = \log\frac{\tau}{\tau_o} - \log\frac{\Gamma\left(\alpha+\frac{1}{\tau}\right)}{\Gamma\left(\alpha_o+\frac{1}{\tau_o}\right)} - \frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)} + \left(\frac{\lambda}{\lambda_o}\right)^{\tau_o}\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)} - \alpha\tau\log\frac{\lambda}{\lambda_o} - \log\frac{\lambda}{\lambda_o} +$$
$$\left(\frac{\alpha\tau}{\tau_o} - \alpha_o\right)\left(\tau_o\log\lambda - \tau_o\log\lambda_o + \frac{\tau_o}{\tau}\psi\left(\alpha+\frac{1}{\tau}\right)\right)$$

$$K(g\|g_o) \quad = \log\frac{\phi_\tau}{\phi_\lambda^{\alpha\phi_\tau}} - \log\frac{\Gamma\left(\alpha+\frac{1}{\tau}\right)}{\Gamma\left(\alpha_o+\frac{1}{\tau_o}\right)} - \frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)} + \mu(\alpha,\phi_\tau,\phi_\lambda) -$$
$$\frac{1}{\tau_o}\log\phi_\lambda + (\alpha\phi_\tau - \alpha_o)v(\alpha,\phi_\tau,\phi_\lambda) \qquad (7.6)$$

where $\phi_\tau = \tau/\tau_o$, $\phi_\lambda = (\lambda/\lambda_o)^{\tau_o}$, $\mu(\alpha,\phi_\tau,\phi_\lambda)$ is the first moment and $v(\alpha,\phi_\tau,\phi_\lambda)$ is the geometric mean of a SBGG distribution with parameters $(\alpha,\phi_\tau,\phi_\lambda)$. The discrimination information of SBGG distribution is a function of ratio of the scales ϕ_λ, the ratio of the weibull shape parameter ϕ_τ, and function of the gamma shape parameters $\Gamma\left(\alpha+\frac{1}{\tau}\right)$ and $\Gamma\left(\alpha_o+\frac{1}{\tau_o}\right)$.

Some special cases

Case (I): If $\phi_\tau = \tau$ in (7.6), then the discrimination information between $SBGG(\alpha,\tau,\lambda)$ and size biased gamma $SBG(\alpha_o,\lambda_o)$ is given by

$$K(SBGG\|SBG) = -\log\Gamma\left(\alpha+\frac{1}{\tau}\right) - \log\lambda + \log\tau + \alpha\psi\left(\alpha+\frac{1}{\tau}\right) - \frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)} + \log\Gamma(\alpha_o+1) +$$
$$\alpha_o\log\lambda_o + \log\lambda_o - \alpha_o\log\lambda - \frac{\alpha_o}{\tau}\psi\left(\alpha+\frac{1}{\tau}\right) + \frac{\lambda}{\lambda_o}\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}$$

Case (II): When $\alpha_o = 1$ in (7.6), the discrimination information between $SBGG(\alpha,\tau,\lambda)$ and size biased weibull $SBW(\tau_o,\lambda_o)$ can be obtained as

$$K\left(SBGG\|SBW\right)=-\log\Gamma\left(\alpha+\frac{1}{\tau}\right)-\log\lambda+\log\tau+\alpha\psi\left(\alpha+\frac{1}{\tau}\right)-\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}-\log\tau_o+$$

$$\log\Gamma\left(1+\frac{1}{\tau_o}\right)+\tau_o\log\lambda_o+\log\lambda_o-\tau_o\log\lambda-\frac{\tau_o}{\tau}\psi\left(\alpha+\frac{1}{\tau}\right)+\left(\frac{\lambda}{\lambda_o}\right)^{\tau_o}\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}$$

Case (III): Put $\phi_\tau=\tau$ and $\alpha_o=1$ in (7.6), the discrimination information between $SBGG(\alpha,\tau,\lambda)$ and size biased exponential $SBE(\lambda_o)$ can be written as

$$K\left(SBGG\|SBE\right)=-\log\Gamma\left(\alpha+\frac{1}{\tau}\right)-\log\lambda+\log\tau+\alpha\psi\left(\alpha+\frac{1}{\tau}\right)-\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}$$

$$-\log\lambda-\frac{1}{\tau}\psi\left(\alpha+\frac{1}{\tau}\right)+2\log\lambda_o+\frac{\lambda}{\lambda_o}\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}$$

Case (IV): Let $\phi_\tau=\tau/2$ and $\alpha_o=2\alpha$ in (7.6), the discrimination information between $SBGG(\alpha,\tau,\lambda)$ and size biased generalized normal $SBGN(\alpha_o,\lambda_o)$ is given as

$$K\left(SBGG\|SBGN\right)=-\log\Gamma\left(\alpha+\frac{1}{\tau}\right)-\log\lambda+\log\tau+\alpha\psi\left(\alpha+\frac{1}{\tau}\right)-\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}-\log 2+$$

$$\log\Gamma\left(\alpha_o+\frac{1}{2}\right)+2\alpha_o\log\lambda_o+\log\lambda_o-2\alpha_o\log\lambda-$$

$$\frac{2\alpha_o}{\tau}\psi\left(\alpha+\frac{1}{\tau}\right)+\left(\frac{\lambda}{\lambda_o}\right)^2\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}$$

6. Data transformation

Size biased generalized gamma distribution illustrates some measures in terms of data transformation and is closed under power transformation. Using (2.2), the discrimination information between $Y \sim SBGG(\alpha,\tau,\lambda)$ and $Y^s \sim SBGG_s\left(\alpha,\frac{\tau}{s},\lambda^s\right)$ is given by

$$K(g\|g_s)=\int g(y)\log g(y)dy-\int g(y^s)\log g(y^s)dy$$

$$=-H(g)+H(g_s) \tag{6.1}$$

$$H(g_s)=-E(\log g(y^s))=-E\left[\log\left\{\frac{\tau}{s\Gamma\left(\alpha+\frac{1}{\tau}\right)\lambda^{s(\alpha\tau+1)}}y^{\frac{\alpha\tau}{s}}e^{-\left(\frac{y}{\lambda^s}\right)^{\frac{\tau}{s}}}\right\}\right]$$

$$=-E\left[\log\tau-\log s-\log\Gamma\left(\alpha+\frac{1}{\tau}\right)-s(\alpha\tau+1)\log\lambda+\frac{\alpha\tau}{s}\log y-\left(\frac{y}{\lambda^s}\right)^{\frac{\tau}{s}}\right]$$

$$=-\log\tau+\log s+\log\Gamma\left(\alpha+\frac{1}{\tau}\right)+s(\alpha\tau+1)\log\lambda-\frac{\alpha\tau}{s}E(\log y)+\frac{1}{\lambda^\tau}E\left(y^{\frac{\tau}{s}}\right)$$

$$=-\log\tau+\log s+\log\Gamma\left(\alpha+\frac{1}{\tau}\right)+s\alpha\tau\log\lambda+s\log\lambda-\frac{\alpha\tau}{s}\log\lambda-\frac{\alpha}{s}\psi\left(\alpha+\frac{1}{\tau}\right)$$
$$+\lambda^{\frac{\tau}{s}-\tau}\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)} \tag{6.2}$$

Using (3.7) and (6.2) in (6.1), the information effect of transformation is obtained as

$$K(g\|g_s)=\log s-\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}+\alpha\psi\left(\alpha+\frac{1}{\tau}\right)+\alpha\tau\log\lambda-\frac{\alpha\tau}{s}\log\lambda-\frac{\alpha}{s}\psi\left(\alpha+\frac{1}{\tau}\right)+\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}\lambda^{\frac{\tau}{s}-\tau}$$

$$=\log s+\frac{\Gamma(\alpha+2)}{\Gamma\left(\alpha+\frac{1}{\tau}\right)}\left[\frac{\mu_{\tau/s}(\alpha,\tau,\lambda)}{\mu_\tau(\alpha,\tau,\lambda)}-1\right]-\alpha\left[v_{\tau/s}(\alpha,\tau,\lambda)-v_\tau(\alpha,\tau,\lambda)\right] \tag{6.3}$$

Equation (6.3) is the ratio of the power means and the difference between the geometric means of the transformed and original variables.

7. Log-likelihood ratio statistic

The likelihood function of SBGGD is given in (4.2) and (4.9) gives $l(\hat{\alpha},\hat{\tau},\hat{\lambda}) = -n\hat{H}(g)$, the log-likelihood ratio statistic for the SBGGD is

$$-2\log\left[\frac{g(y|\alpha_o,\tau_o,\lambda_o)}{g(y|\hat{\alpha},\hat{\tau},\hat{\lambda})}\right] = -2\left[\frac{-nH(\hat{g}_o)}{-nH(\hat{g})}\right] = 2n(H(\hat{g}_o) - H(\hat{g})) \quad (7.1)$$

If $\alpha_o = 1$, (7.1) redues to size biased weibull, when $\alpha_o = \tau_o = 1$, (7.1) reduces to size biased exponential and if $\alpha_o = 2\alpha$, $\tau_o = 2$, (7.1) reduces to size biased generalized normal distribution.

7.2. Akaike and Bayesian information criterion

Informative criteria provide an attractive basis for model selection. For this the research evaluated the performance of the two commonly used model selection criteria Akaike and Bayesian informative criteria (AIC & BIC). The BIC was introduced by Schwarz [18] as a competitor to the AIC [1] & [2]. The AIC is a measure of the relative quality of a statistical model for a given set of data. AIC offers a relative estimate of the information lost when a given model is used to represent the process that generates the data and deals with the trade-off between the goodness of fit of the model and the complexity of the model. Given any two estimated models, the model with the lower value of AIC is the one to be preferred. BIC is a criteria for medel selection among a finite set of models, the model with the lowest BIC is preferred and it is very closely related to the AIC. The penalty term of BIC is more stringent than the penalty term of AIC.

The general form of AIC and BIC are given by

$$AIC = 2K - 2\log(L(\hat{\theta}))$$

$$BIC = K\log n - 2\log(L(\hat{\theta}))$$

The AIC and BIC for the SBGGD using (4.9) are given as follows

$$= 2K + 2nH(\hat{g})$$

$$= K \log n + 2nH(\hat{g})$$

where K is the number of parameters to be estimated and n is the sample size .

8. Conclusion.

Since the Size Biased Generalized Gamma Distribution is closed under power transformation. This paper defined the new form of SBGGD using the power transformation technique. Using power and lograthemic moments entropy representation and its estimation of SBGGD have been studied. The entropy of subfamilies for SBGGD has also been discussed. Various information measures of SBGGD were investigated including Kullback-Leibler discrimination with subfamilies, data transformation, Log-likelihood ratio statistic, Akaike and Bayesian information criterion.

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