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A HIGHER ODER POLYNOMIAL FOR OPTIMAL PRICING AND REPLENISHMENT STRATEGIES FOR A RETAIL/DISTRIBUTION SYSTEM

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ABSTRACT

We consider a distribution system in which a supplier distributes a product to N competing retailers. The demand rate of each retailer depends on all of the retailers' prices, or alternatively, the price each retailer can charge for its product depends on the sales volumes targeted by all of the retailers. The supplier replenishes his inventory through orders (purchases, production runs) from an outside source with ample supply. Carrying costs are incurred for all inventories, while all supplier orders and transfers to the retailers incur fixed and variable costs. All retailer prices, sales quantities and the complete chain-wide replenishment strategy are determined by a single decision maker, e.g., the supplier. Using a polynomial of order 3 as annual cost of managing retailers account, we show that profit Π lies within two extremes Π_1 and Π_2, where Π_1 is the minimum profit and Π_2 is the maximum profit.

***Keywords*:** Stock outs; Inventory/Production; marketing/pricing policies; Marketing; Channels of distribution; retailing; pricing.

INTRODUCTION

Generally a firm goes into the business of production in order to make a maximum profit. So if a firm as a constant function C, so that it cost $C(q)$ to produce q units of its product and suppose that the product can be sold at a price $p(q)$ per unit, the firms revenue from producing q units (Antony and Biggs 2000) is

$$R(q) = q(p(q))$$

and its profit

$$\Pi(q) = R(q) - C(q)$$

In their attempt to improve or optimize aggregate performance, many supply chains increasingly investigate and compare their performance under centralized and decentralized decision making. In a decentralized system, each chain member optimizes his own profit function. The challenge therefore consists of structuring the costs and rewards of all of the chain members so as to align their objectives with aggregate supply-chain-wide profits. Such a cost and reward structure is referred to as a coordination mechanism. In the absence of retailer competition, discounts based on the annual sales volume arise only in the presence of account management costs, as demonstrated in Chen et al. (2001). On the other hand, in the presence of retailer competition, such discounts are required even if no account management costs prevail. The coordination mechanism thus provides an economic rationale, within the context of a model with complete information and symmetric bargaining power for all retailers, for wholesale prices to be discounted on the basis of annual sales volumes, one of the most prevalent forms of price discount schemes (see e.g., Brown and Medoff 1990, Stein and El-Ansary 1992, and Munson and Rosenblatt 1998). We analyze the performance of the system, assuming either that the supplier has the market power to specify the nonlinear wholesale pricing scheme, or that a polynomial wholesale price is chosen so as to optimize the supply-chain-wide profits. The marketing literature on channel coordination focuses on pricing decisions. McGuire and Staelin (1983) consider the special case of two identical retailers, competing in price space under linear procurement costs. These authors assume that the two retailers are supplied by two different manufacturers which are either vertically integrated with their retailer or not. Raju and Zhang (1999) analyze another variant of our model with one dominant retailer capable of singlehandedly setting the retail price which is adopted by all other retailers in the market.

Under a linear cost structure, the authors show that with a linear wholesale pricing scheme, perfect coordination requires that double marginalization be avoided. The marketing literature has thus restricted itself to the simplest of cost structures, i.e., to the case of linear costs. As summarized above and demonstrated below, more complex, yet basic, operational cost structures such as those arising under inventory carrying and fixed distribution costs introduce additional and essential complexities to the challenge of designing appropriate coordination mechanisms. This point has been brought out in an emerging stream of operations management papers. We refer to Chen et al. (2001) and, Bernstein and Federgruen (2003) for a review of the literature on models with exogenously given, deterministic demand processes. (This stream of papers appears to have originated with Crowther (1964), examining quantity discounts from both the buyer's and the seller's perspective. Lee and Whang (1996), Chen (1999) , Cachon (1999) and Zipkin (1999) have developed perfect coordination schemes for a stochastic version of our model with a single retailer facing an exogenously given demand process and in the absence of fixed costs for deliveries from the supplier to the retailers. Weng (1995) is one of the first attempts to treat the retailers' demand rates as endogenous variables to be determined by a careful balancing of revenue as well as cost considerations. This model considers the special case of a single retailer or multiple, but identical and noncompeting retailers. The author asserts that an order quantity discount plus a periodic franchise fee suffice to achieve perfect coordination. This assertion, however, has not been substantiated, as Boyaci and Gallego (1997) pointed out. Chen et al. (2001) address the centralized and the decentralized versions of our supply-chain model, in the absence of the retailers competing in price or quantity space, i.e., when each retailer's sales volume is a function of his own price only. See Munson and Rosenblatt (1998), Boyaci and Gallego (1997), Cachon (1999), Lariviere (1999), and Tsay et al. (1999) for additional reviews of the operations management literature related to channel coordination with noncompeting retailers. Other researchers have also worked on optimal pricing, Bernstein and Federgruen (2003) has a good review of some of this work.

MODEL AND NOTATION

We consider a continuous-review inventory model with a price-sensitive demand. The objective is to determine the inventory replenishment and pricing decisions that strike a balance between the sales revenue and the cost for holding and replenishing inventory over time, so as to maximize the expected long-run average or discounted profit. Consider a distribution system with a supplier distributing a single product or closely substitutable products to N retailers. The retailers sell their product to the final consumer. The supplier replenishes his inventory from a source with ample supply. All demands and all retailer orders must be satisfied without incurring any stockouts. We assume that all orders are received instantaneously upon placement. Positive but deterministic lead-times can be handled by a simple shift in time of all desired replenishment epochs. Thus, let

p_i = retail price charged by retailer i, and

q_i = consumer demand for retailer $i's$ product.

The two sets of variables may be related to each other via the (direct) demand functions

$$q_i = d_i(p_i,..., p_N), i = 1,.., N, \qquad (1)$$

or the inverse demand functions

$$q_i = f_i(q_i,..., q_N), i = 1,.., N. \qquad (2)$$

We assume that all demand functions are downward sloping, a property almost invariably satisfied, with the exception of rare luxury, or Veblen goods:

$$\frac{\partial d_i}{\partial p_i} < 0, \; i = 1,..., N \qquad (3)$$

To simplify some of the results, we shall consider the case where demand functions are linear. In particular,

$$d_i(p) = a_i - b_i p_i + \sum_{j \neq i} \beta_{ij} p_j \qquad (4)$$

with $a_i > 0,\; b_i > 0,\; i = 1,..., N$

Because the retailer products are substitutes, we have, by a common definition going back to Samuelson (1947), that

$$\beta_{ij} \geq 0 \text{ for all } i \neq j \qquad (5)$$

We assume in addition that the matrix B, with $B_{ii} = -b_i$ and $B_{ij} = \beta_{ij}$ for all $i \neq j$, is nonsingular, so that the inverse demand functions exist and are linear as well, i.e.,

$$f_i(q) = \hat{a}_i - \hat{b}_i\, p_i + \sum_{j \neq i} \hat{\beta}_{ij}\, q_j, \; i = 1,..., N \qquad (6)$$

Moreover, we would like the inverse demand functions to be downward sloping and products to be substitutes in terms of the inverse demand functions as well (see Vives 2000):

$$\hat{b}_i > 0,\; \hat{\beta}_{ij} > 0 \text{ for all } i \neq j,\; i = 1,..., N \qquad (7)$$

Unfortunately, (7) is not necessarily implied by the corresponding properties (3) and (5) for the direct demand functions. (see Bernstein and Federgruen (2003) for additional explanation)

Bernstein and Federgruen (2003) considered a two – echelon distribution system in which a supplier distributed a product to N competing retailers. The demand rate of each retailer depends on the retailers' price, or alternatively, the price each retailer can charge for its product depends on the sales volume targeted by all the retailers. The solution was first characterised by the centralised system in which all retailers' prices, sales quantities and the complete chain wide replenishment strategy were determined by a single decision maker. The system wide profit, Π (Chen et al. 2001) is given by

$$\Pi = -\frac{k_0}{T_0} + \sum_{i=1}^{\infty} G_i(d_i, T_i, T_0) \qquad (8)$$

where

$$G_i(d_i, T_i, T_0) = (p_i(d_i) - c_0 - c_i)d_i - \psi(d_i) - \frac{k_i}{T_i} - \frac{1}{2}h_0 d_i \max(T_0, T_i) - \frac{1}{2}h_i d_i T_i \qquad (9)$$

T_i replenishment interval for retailer i, $i = 1,...N$

T_0 replenishment interval for supplier

p_i retail price charged by retailer i

$d_i(p_i)$ annual consumer demand in the market served by retailer i, a strictly decreasing function of p_i

k_0 fixed cost incurred for each delivery to the supply i

k_i fixed cost incurred for each delivery to retailer i, $i = 1,...N$

h_0 basic annual holding cost per unit in inventory of retailer i

h_i incremental annual holding cost per unit in inventory of retailer i

c_0 fixed cost per unit ordered by retailer

c_i variable cost per ordered by retailer i

$\psi_i(d_i)$ annual cost for managing retailer i's account.

3. Higher order polynomial model

Chen et al. (2001) investigated equation (8) when

$$\psi = f + e_i d_i \tag{10}$$

$$p_i(d_i) = a_i - b_i d_i \tag{11}$$

Where f, e_i, a_i, b_i and d_i are positive numbers

Lemma (Chen et al. (2001)):

$T_i(d_i)$ is decreasing in d_i for $i = 1, 2 \ldots N$

$d_i(T_i)$ is decreasing in T_i for $i = 1, 2 \ldots N$

Here ψ is a polynomial of order one in d_i and is concave downward in d_i with a maximum profit at

$$d_i = \frac{1}{2b_i}\left(a_i - c_0 - c_i - e_i - \frac{1}{2}h_0 d_i \max(T_0, T_i) - \frac{1}{2}h_i d_i T_i\right) \tag{12}$$

In this paper, following (Adeniran 2004) we assume

$$\psi = f + e_i d_i + a_i d_i^2 + b_i d_i^3 \tag{13}$$

and retain equation (11)

Theorem

There exist $d_i^{\bullet}$ and $d_i^{\bullet\bullet}$ where $\Pi(d_i^{\bullet})$ is maximum and $\Pi(d_i^{\bullet\bullet})$ is minimum

Proof

$$\frac{\partial \Pi}{\partial d_i} = \left\{ p_i'(d_i)d_i + p_i(d_i) - c_0 - c_i - \psi'(d_i) - \frac{1}{2}h_0 \max(T_0, T_i) - \frac{1}{2}h_i T_i \right\} \tag{14}$$

$$\frac{\partial^2 \Pi}{\partial d_i^2} = \left\{ p_i''(d_i)d_i + 2p_i'(d_i) - \psi''(d_i) \right\} \tag{15}$$

Setting $\frac{\partial \Pi}{\partial d_i} = 0$, we obtain

$$d_i^{\bullet} = \alpha_i + \beta_i \tag{16}$$

$$d_i^{\bullet\bullet} = \alpha_i - \beta_i \tag{17}$$

where

$$\alpha_i = -(b_i + a_i) \tag{18}$$

and

$$\beta_i = \sqrt{4(b_i + a_i)^2 + 12b_i \begin{pmatrix} a_i - c_0 - c_i - e_i \\ -\frac{1}{2}h_0 \max(T_0, T_i) - \frac{1}{2}h_i T_i \end{pmatrix}} \tag{19}$$

Putting (16) and (17) into $\frac{\partial^2 \Pi}{\partial d_i^2}$, we obtain

$$\frac{\partial^2 \Pi}{\partial d_i^2}(d_i^{\bullet}) < 0 \tag{20}$$

$$\frac{\partial^2 \Pi}{\partial d_i^2}(d_i^{\bullet\bullet}) > 0 \tag{21}$$

This completes the proof.

PRICING AND REPLENISHMENT POLICIES

Assume replenishment is instantaneous, i.e., with zero lead time. We further assume that all orders (demand) will be supplied immediately upon arrival; i.e., no back-order is allowed, or there is an infinite back-order cost penalty.

The replenishment follows a continuous-review, order-up-to policy. Specifically, whenever the inventory level drops to zero, it is brought up to S instantaneously via a replenishment, where S is a decision variable.

We shall refer to the time between two consecutive replenishments as a cycle.

We adopt the following dynamic pricing strategy. Let $N \geq 1$ be a given integer, and let $S = S_0 > S_1 > S_2 > \ldots S_{N-1} > S_N = 0$. Immediately after a replenishment at the beginning of a cycle, price p1 is charged until the inventory drops to S_1, price p_2 is then charged until the inventory drops to S_2, ..., and finally when the inventory level drops to S_{N-1}, price p_N is charged until the inventory drops to $S_N = 0$, when another cycle begins. The same pricing strategy applies to all cycles. For simplicity, we set $S_n = \dfrac{S(N-n)}{N}$. That is, we divide the full inventory of S units into N equal segments, and price each segment with a different price as the inventory is depleted by demand.

In summary, the decision variables are: (S, p), where $S \in \Re_+$, and $p = (p_1, \ldots, p_N) \in \mathrm{P}^N$. Within a cycle, we shall refer to the time when the price p_n is applied as period n.

CONCLUDING REMARKS

In this paper, we have compared the optimal performance of the supply chain operating under given types of wholesale pricing schemes. It is easiest to characterize the performance of the chain under a simple linear wholesale pricing scheme, characterized by an arbitrary vector of constant wholesale prices. There are, however, a number of important differences between the equilibrium behaviour of the retailers under price and quantity competition. If the retailers compete in quantity space, each adopts a retail price that is larger than its equilibrium price under price competition. Larger sales volumes, under price competition, can only be guaranteed in special cases, e.g., when the retailers are identical. In the case of price competition, an equilibrium price vector can be found with the help of the simple iterative tatonnement scheme (Bernstein and Federgruen 2003). Unfortunately, perfect coordination cannot be achieved under any linear wholesale pricing scheme. To achieve perfect coordination, a nonlinear wholesale pricing scheme is required. We derive such a scheme and analyse it in this present work. Our coordination mechanism therefore provides an economic rationale, within the context of a model with complete information and symmetric bargaining power, for wholesale prices to be discounted on the basis of annual sales volumes. (This type of discount scheme is most prevalent in practice). The wholesale price charged to retailer i under the coordinating scheme equals the average cost (per unit of sales) of all cost components incurred by the supplier that are directly related to retailer i's sales, plus a markup. While linear wholesale pricing schemes fail to achieve perfect coordination, they appear to allow for modest gaps with respect to the first-best or centralized solution. (The gaps are modest compared to those arising under Stackelberg solutions).

Our models can be extended to allow backlogging, in which case the replenishment takes the form of a $(-s, S)$ policy: a replenishment order is issued whenever the backlog has reached s, to bring the inventory level back to S. (Hence, the replenishment quantity is $S + s$. Assume zero leadtime as before.) The pricing policy is modified accordingly: equally divide S and s into N and M (positive integers) segments, respectively, such that

$$\begin{aligned} S &= S_0 > S_1 > S_2 > \ldots S_{N-1} > S_{N-1} \\ &= 0 > S_{N+1} > \ldots S_{N+M} \\ &= -s, \end{aligned}$$

and apply price p_n until the net inventory (inventory on hand net the backlogs) falls to S_n, $n = 1, \ldots N + M$.

Also, the optimal pricing will satisfy the monotonicity:

$$p_1^* \leq p_2^* \geq \ldots \leq p_N^*, \quad p_{N+1}^* \geq p_{N+2}^* \geq \ldots \geq p_{N+M}^*$$

That is, the optimal prices increase as the on-hand inventory is depleted, and decrease as the backlog increases.

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